The Global Electroweak Fit in the Standard Model

Robert Clare
Massachusetts Institute of Technology, Cambridge, MA 02139

A brief summary of the electroweak measurements is presented, followed by fits of the measurements within the framework of the Standard Model. The data are in very good agreement with the Standard Model predictions, and can be used to probe purely weak radiative corrections. From the measurements of the radiative corrections, the upper limit on the mass of the Higgs boson can be set at 230 GeV at 95% confidence level.

I. INTRODUCTION

The measurements performed at LEP, SLC and Tevatron on the properties of the Z and W bosons and the top quark have shed a significant amount of light on the Standard Model. The precision of the measurements has allowed a test of the Standard Model beyond the tree level. Effects of weak radiative corrections can be measured, and from these measurements information on the Higgs sector of the Standard Model can be inferred.

The relevant measurements are the mass and width of the Z boson which are determined from measurements of the production cross-section at the Z pole, and the couplings of the Z boson to fermions, determined from asymmetries and cross-sections. These measurements have been performed at LEP and SLC. Important additional measurements are the mass of the W boson (determined at $\mathrm{p\overline{p}}$ colliders and LEP), the mass of the top quark (from the Tevatron), and the measurements of the weak mixing angle, $\sin^2\theta_{\mathrm{W}}$, from neutrino-nucleon experiments.

The results discussed here have been updated to include the results presented at Moriond 1999. Most are still preliminary.

II. MEASUREMENTS OF THE Z PROPERTIES

The measurements performed using Z decays can be roughly divided into two classes: cross-sections and asymmetries.

The cross-section is essentially the Breit-Wigner of the Z pole convoluted with initial-state radiation (as well as interference between Z and photon exchange). From the full set of LEP data [1–4] (14.8 million hadronic and 1.6 million leptonic Z decays), the following have been determined:

$$m_{\mathrm{Z}} = 91.1867 \pm 0.0021 \text{ GeV}, \tag{1}$$
$$\Gamma_{\mathrm{Z}} = 2.4939 \pm 0.0024 \text{ GeV}, \tag{2}$$
$$\sigma^0_{\mathrm{h}} = 41.491 \pm 0.058 \text{ nb}, \tag{3}$$
$$R_\ell = 20.765 \pm 0.026. \tag{4}$$

The mass and the width are those based on a running-width Breit-Wigner denominator $(s - m^2_{\mathrm{Z}} + is\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}})$ [5]. The hadronic pole cross-section is defined as $\sigma^0_{\mathrm{h}} \equiv \frac{12\pi}{m^2_{\mathrm{Z}}}\frac{\Gamma_{\mathrm{ee}}\Gamma_{\mathrm{had}}}{\Gamma^2_{\mathrm{Z}}}$, where Γ_{ee} and Γ_{had} are the partial widths of the Z into electrons and hadrons. The ratio $R_\ell \equiv \Gamma_{\mathrm{had}}/\Gamma_{\ell\ell}$ is the ratio of the partial widths into hadrons and a pair of massless leptons, and is derived assuming lepton universality. This set of parameters was chosen to minimize the experimental correlations.

There have been three significant developments that have taken place over the last year which have affected, or will affect these results. These are: the inclusion of third-order QED radiative corrections [6–8], the treatment of the theoretical uncertainties in the t-channel subtraction [9] for the electron final state as a common systematic error, and the reduction of the theoretical error on the luminosity [10].

The main effect of QED radiative corrections is to reduce the effective center of mass energy. Thus, the corrections are asymmetic around the Z peak. The inclusion of third-order corrections leads to a shift in the measured mass and width of the Z boson. Last summer it was thought that some calculations included these corrections while others did not, and it was expected that the two sets would then agree better when the corrections were introduced. This was not the case, and caused some concern. Therefore, the full effect of the correction was included as a systematic error for the results prepared for the summer conferences. Since then, it was determined that none of the calculations included these corrections, so the apparent problem has been clarified. Thus, for the next round of averages (which will be performed once the four LEP collaborations publish the final lineshape results) this correction will be included correctly. We expect a shift of about 0.5 MeV in m_{Z} and Γ_{Z} and about 0.05% in σ_{h}^0, with a corresponding decrease in the errors.

The treatment of theoretical error on the t-channel subtraction for the electron final states is also new. This error source has been taken into account by the experiments in previous fits. New is the treatment of this error as a common systematic error. This slightly increased the overall error on $A_{\mathrm{FB}}^{0,\mathrm{e}}$ and R_{e}, and has introduced a correlation between them. This error source is still under study, and may change in the future.

The final new development is the reduction of the theoretical error on the luminosity. The new calculation was not yet ready at the time of the last round of fits, so is not yet included. Once it is included, the error on σ_{h}^0 and N_ν, the number of light neutrino generations, will be significantly reduced.

The other class of Z measurements are the asymmetries. The forward-backward asymmetry is defined as $A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_{\mathrm{tot}}}$ where σ_F and σ_B are the cross-sections for events with the out-going fermion going in the direction of the incoming electron or positron, respectively. The asymmetries are corrected for effects of initial-state radiation, and, in the case of electron final states, for t-channel exchange and s-t interference. The resulting pole asymmetries, $A_{\mathrm{FB}}^{0,\mathrm{f}}$ can be expressed in terms of the vector and axial-vector coupling constants:

$$A_{\mathrm{FB}}^{0,\mathrm{f}} = \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_{\mathrm{f}}; \tag{5}$$

$$\mathcal{A}_{\mathrm{f}} = \frac{2 g_{V\mathrm{f}} g_{A\mathrm{f}}}{g_{V\mathrm{f}}^2 + g_{A\mathrm{f}}^2}. \tag{6}$$

From these coupling constants, the effective weak mixing angle is defined:

$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = \frac{1}{4}(1 - g_{V\ell}/g_{A\ell}). \tag{7}$$

Parity violation both in the production of Z bosons as well as in their decay produces polarized final states even if the incoming electron and positrons are unpolarized. At LEP, only the $\tau^+\tau^-$ final states can be analyzed, using the tau decay products as a polarimeter. The angular dependence of the polarization is given by

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau(1+\cos^2\theta) + 2\mathcal{A}_{\mathrm{e}}\cos\theta}{1+\cos^2\theta + 2\mathcal{A}_\tau\mathcal{A}_{\mathrm{e}}\cos\theta}, \tag{8}$$

where θ is the tau production angle. From a measurement of the tau polarization, $\mathcal{A}_\tau$ and $\mathcal{A}_{\mathrm{e}}$ can be independently determined. In addition, as the measurement is linear in $\mathcal{A}_\tau$ and $\mathcal{A}_{\mathrm{e}}$, the relative sign of $g_{A\ell}$ and $g_{V\ell}$ can also be determined.

A very similar physics result has been obtained by the SLD collaboration at the SLC [11]. In their case, the incoming electron beam is given either right- or left-handed polarization. A straight-forward measurement of the hadronic cross-section for right and left polarized electrons is then used to determine $\mathcal{A}_{\mathrm{e}}$:

$$A_{\mathrm{LR}} = \frac{1}{\mathcal{P}_{\mathrm{e}}}\frac{\sigma_L - \sigma_R}{\sigma_{\mathrm{tot}}} = \mathcal{A}_{\mathrm{e}}. \tag{9}$$

The physics is the same as in tau polarization. However, at SLC the electron beam is polarized to 77%, whereas the natural polarization of Z decays is only 14%.

Using the results of the lepton asymmetries, the tau polarization, the $A_{\rm LR}$ measurement, and the determinations of $\Gamma_{\ell\ell}$, the values of the vector and axial-vector coupling constants for leptons can be extracted ($\Gamma_{\ell\ell} \propto g_{V\ell}^2 + g_{A\ell}^2$). The result, assuming lepton universality, is:

$$g_{V\ell} = -0.03753 \pm 0.00044, \tag{10}$$

$$g_{A\ell} = -0.50102 \pm 0.00030. \tag{11}$$

This is shown graphically in Fig. 1. As can be seen from the Figure, the latest SLD results are in much better agreement with the LEP results than they were in 1997 [12].

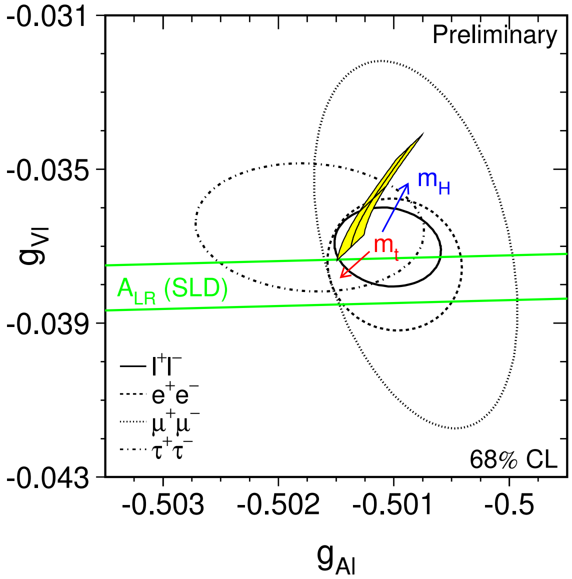

FIG. 1. The 68% contours in the $g_{V\ell}$, $g_{A\ell}$ plane. The solid ellipse is for the assumption of lepton universality.

The analysis of hadronic final states containing heavy quarks, especially b quarks, also yields important information. Because of the large t-b mass splitting, the decay $\rm Z \to b\overline{b}$ has large vertex corrections involving loops with t quarks. The ratio $R_{\rm b} \equiv \Gamma_{\rm b\bar{b}}/\Gamma_{\rm had}$ therefore has a unique sensitivity to $m_{\rm t}$. In addition, because the b quark has charge 1/3, the forward-backward $\rm b\overline{b}$ asymmetry is very sensitive to $\mathcal{A}_{\rm e}$, and thus $\sin^2\theta_{\rm eff}^{\rm lept}$.

A couple of years ago, the measurement of $R_{\rm b}$ was the cause of some concern, as it was several standard deviations away from its Standard Model prediction. However, with the addition of significantly more data, greatly improved analysis techniques, and some new measurements of auxiliary branching ratios, the situation has changed considerably. This is shown in Fig. 2, which shows the evolution of the measurements over the past 4 years.

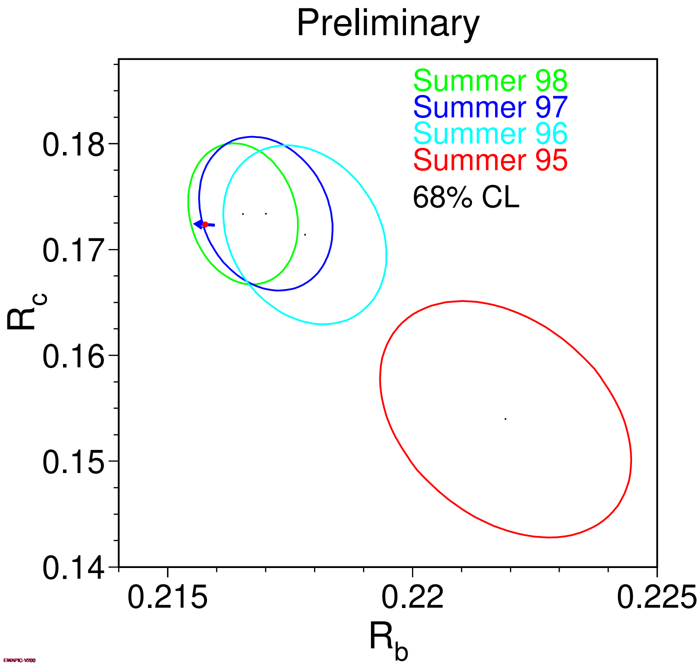

FIG. 2. The 68% contours for the measurements of R_{b} and R_{c}. The arrow represents the Standard Model prediction for a top quark mass of 175 ± 5 GeV.

The summary of all the asymmetry results from the Z data, interpreted in terms of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ is shown in Fig. 3. Also here there has been a significant change since last year. The χ^2 per degree of freedom for the average is 7.7/6, which is quite acceptable. This has improved considerably over the last couple of years, although it is true that the two most precise values, those coming from A_{LR} and $A_{\mathrm{FB}}^{0,\,\mathrm{b}}$ are 2.4 standard deviations apart.

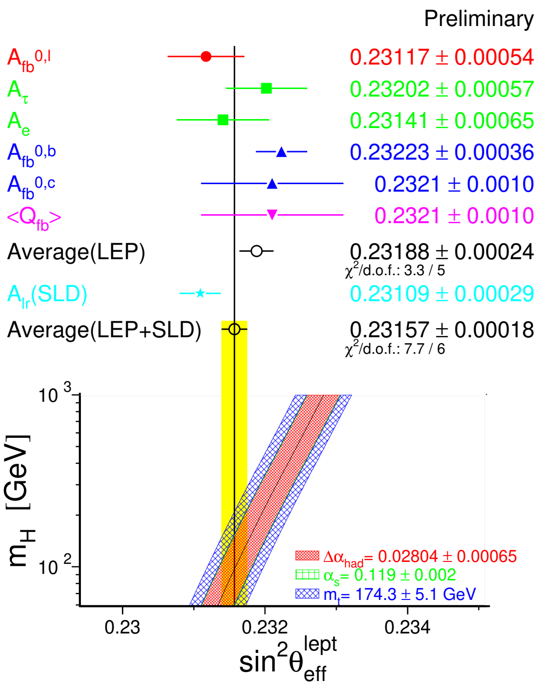

FIG. 3. The measurement of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ from asymmetry measurements at LEP and SLD.

III. THE W BOSON

Since 1996, LEP has been running at energies in excess of 160 GeV, *i.e.*, above the threshold for the production of a pair of W bosons. By now, each experiment has accumulated aproximately 3500 W-pair events, enabling a precise measurement of the properties of the W boson. For our purposes, we will look only at the mass of the W, since it has the largest impact on the mass of the Higgs boson.

The W mass is determined through the reconstruction of its decay products. There are two main channels considered: when both W bosons decay hadronically (hadronic decays), and when one decays leptonically and the other hadronically (semileptonic decays). The third channel, where both W's decay leptonically is currently not used. With the large statistics now, the effects of systematic errors become important. The LEP experiments have concentrated on five types of systematic errors: errors uncorrelated between channels and experiments; those correlated within the measurements of an experiment but uncorrelated with other experiments, such as detector calibrations; errors associated with final state interactions (FSI) which affect only the hadronic channel; errors from ISR and fragmentation which are correlated between channels and experiments; and finally the LEP energy scale, correlated between experiments and different data-taking years.

The individual measurements of the W mass from the four LEP experiments [13] is shown in Figure 4. The difference between the hadronic and semileptonic channels is 152 ± 74 MeV (not including FSI effects). Combining the two channels, and using also the previous mass deterimination from the cross-section measurements at 161 GeV results in $m_{\rm W} = 80.350 \pm 0.056$GeV.

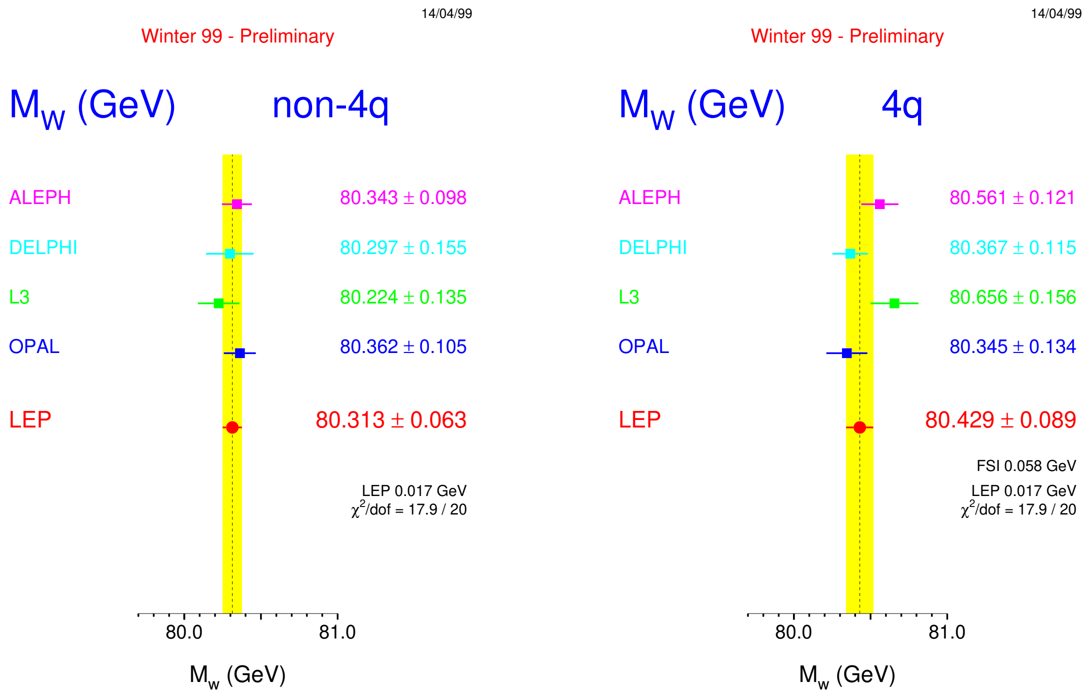

FIG. 4. The measurements of the W mass at LEP.

IV. THE GLOBAL FIT

Within the framework of the Standard Model, the effects of radiative corrections can be used to infer information about unobserved particles. A careful comparison of the theory with the measurements can thus yield interesting information about the structure of the radiative corrections as well as the mass of the Higgs boson. This is especially true since the discovery of the top quark at the Tevatron. The measurements used in the following Standard Model fits are shown in Table I. In addition to the measurements at LEP and SLD, measurements of G_{F} from muon decay, m_{t} [14] and m_{W} [15] from $\mathrm{p\overline{p}}$ colliders, $\sin^2\theta_{\mathrm{W}}$ from neutrino-nucleon experiments [16], and the determination of $\alpha(m_{\mathrm{Z}}^2)$ [17] from low energy $\mathrm{e^+e^-}$ interactions are also used.

In the Standard Model, relations exist between all masses. Thus, in addition to extracting limits on the Higgs mass, one can test the consistency of the Standard Model. As an example, indirect measurements of m_{t} and m_{W} using all the data except the direct LEP and $\mathrm{p\overline{p}}$ measurements can made. The results are

$$m_{\mathrm{t}} = 157^{+9}_{-8}\ \mathrm{GeV} \tag{12}$$

$$m_{\mathrm{W}} = 80.326 \pm 0.037\ \mathrm{GeV}, \tag{13}$$

which can be compared with the direct measurements of $m_{\mathrm{t}} = 174.3 \pm 5.1$ GeV and $m_{\mathrm{W}} = 80.394 \pm 0.042$ GeV. This is also shown in Fig. 5 where the 68% confidence level contours are given.

As can be seen, the agreement is very good.

Now that the consistency of the data has been checked we can go the next step to investigate the limits on the Higgs mass. In the final fit, all of the data are fit to the Standard Model expectations. This results in

$$m_{\mathrm{t}} = 171.6 \pm 4.9\ \mathrm{GeV} \tag{14}$$

$$m_{\mathrm{H}} = 76^{+79}_{-45}\ \mathrm{GeV}, \tag{15}$$

$$\alpha_s(m_{\mathrm{Z}}^2) = 0.119 \pm 0.003. \tag{16}$$

The χ^2/d.o.f. is a very good 15/15. As pointed out earlier, the dominant Higgs mass dependency is logarithmic, so some care must be taken in interpreting the Higgs mass result.

In order to extract the mass limit, we plot the $\Delta\chi^2$ of the fit as a function of the Higgs mass. This is shown in Fig. 6. In this figure, there are several things that can be noticed:

TABLE I. The experimental input values to the Standard Model fits. Note that many of the results are correlated. Although not shown in the table, the correlation matrices, where appropriate, have been used in the fits. The numbers in brackets give the error in the last digits.

G_{F}	$1.16637(1) \times 10^{-5}$	$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ (A_{LR})	0.23109(29)
$\alpha(m_{\mathrm{Z}}^2)^{-1}$	128.78(90)	R_{b}^0	0.21680(73)
m_{Z} (GeV)	91.1687(21)	R_{c}^0	0.1694(38)
Γ_{Z} (GeV)	2.4939(24)	$A_{\mathrm{FB}}^{0,\,\mathrm{b}}$	0.0991(20)
σ_{h}^0 (pb)	41.491(58)	$A_{\mathrm{FB}}^{0,\,\mathrm{c}}$	0.0712(43)
R_ℓ	20.765(26)	$\mathcal{A}_{\mathrm{b}}$	0.908(27)
$A_{\mathrm{FB}}^{0,\,\ell}$	0.01683(96)	$\mathcal{A}_{\mathrm{c}}$	0.651(30)
$\mathcal{A}_\tau$	0.1431(45)	$\sin^2\theta_{\mathrm{W}}$ νN	0.2255(21)
$\mathcal{A}_{\mathrm{e}}$	0.1479(51)	m_{W} (GeV) $\mathrm{p\overline{p}}$	80.448(62)
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ (Q_{FB})	0.2321(10)	m_{t} (GeV) $\mathrm{p\overline{p}}$	174.3(5.1)
m_{W} (GeV) LEP	80.350(56)		

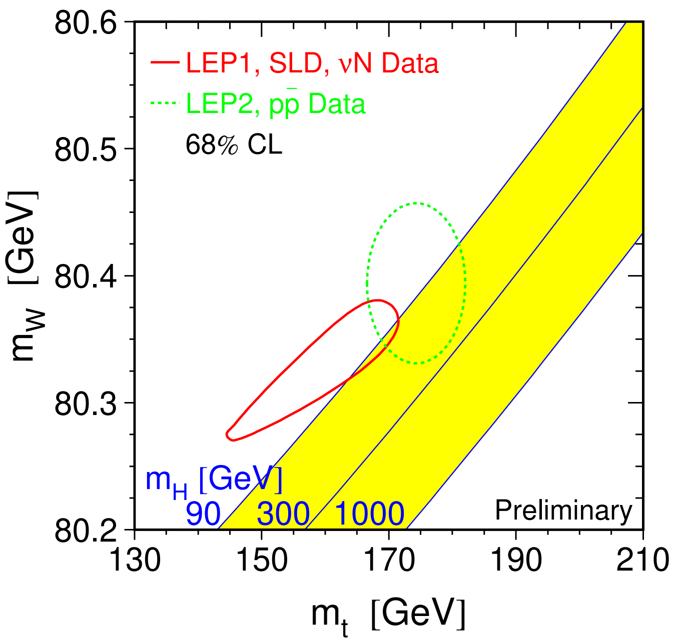

FIG. 5. The 68% C.L. contours for the indirect measurements of m_{W} and m_{t} (solid curve) and the direct measurements (dashed curve). The shaded band shows the Standard Model prediction as a function of the Higgs mass.

- the size of the theoretical uncertainty (grey band) has been reduced significantly in the last couple of years. This is most due to the inclusion of higher order radiative corrections. There has also been a significant amount of work invested by the ZFITTER and TOPAZ0 teams to understand remaining differences.
- The error on $\alpha(m_{\mathrm{Z}}^2)$ gives an important contribution to the mass limit, as can be seen if a value with a smaller error is used (dashed curve).
- The central value of the fit prefers a Higgs mass that has already been excluded by the direct searches (89 GeV) [18].

The 95% C.L. lower limit on the Higgs mass is 230 GeV. This does not take into consideration the direct search limit.

I would like to thank the organizers for a very well run conference. This talk would not have been possible without the support and contributions from my colleagues and friends in the LEP experiments and in the LEP Electroweak Working Group.

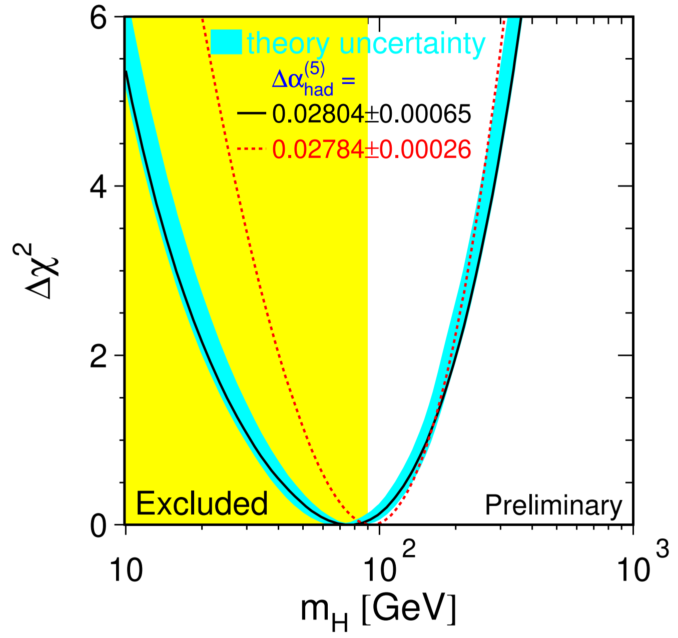

FIG. 6. The $\Delta\chi^2$ curve of the Standard Model fit to the data. See the text for a discussion of the different curves.

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