# Explanations of pulsar velocities

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Several mechanisms based on neutrino oscillations can explain the observed motions of pulsars if the magnetic field in their interiors is of order $10^{14} - 10^{15}$ G.

## I. INTRODUCTION

The proper motions of pulsars [1] present an intriguing astrophysical puzzle. The measured velocities of pulsars exceed those of the ordinary stars in the galaxy by at least an order of magnitude. The data suggest that neutron stars receive a powerful "kick" at birth. Whatever the cause of the kick, the same mechanism may also explain the rotations of pulsars under some conditions [2].

The origin of the birth velocities is unclear. Born in a supernova explosion, a pulsar may receive a substantial kick due to the asymmetries in the collapse, explosion, and the neutrino emission affected by convection [3]. Evolution of close binary systems may also produce rapidly moving pulsars [4]. It was also suggested [5] that the pulsar may be accelerated during the first few months after the supernova explosion by its electromagnetic radiation, the asymmetry resulting from the magnetic dipole moment being inclined to the rotation axis and offset from the center of the star. Most of these mechanisms, however, have difficulties explaining the magnitudes of pulsar spatial velocities in excess of 100 km/s. Although the average pulsar velocity is only a factor of a few higher, there is a substantial population of pulsars which move faster than 700 km/s, some as fast as 1000 km/s [1].

Neutrinos carry away most of the energy, $\sim 10^{53}$ erg, of the supernova explosion. A 1% asymmetry in the distribution of the neutrino momenta is sufficient to explain the pulsar "kicks". A strong magnetic field inside the neutron star could set the preferred direction. However, the neutrino interactions with the magnetic field are hopelessly weak.

Ordinary electroweak processes [6] cannot account for the necessary anisotropy of the neutrino emission [7]. The possibility of a cumulative build-up of the asymmetry due to some parity-violating scattering effects has also been considered [8]. However, in statistical equilibrium, the asymmetry does not build up even if the scattering amplitudes are asymmetric [7,9]. Although some net asymmetry develops because of the departure from equilibrium, it is too small to explain the pulsar velocities for realistic values of the magnetic field inside the neutron star [7,10].

There is a class of mechanisms, however, that can explain the birth velocities of pulsars as long as the magnetic field inside a neutron star is $10^{14} - 10^{15}$ G. These mechanisms [11–14] have some common features. First, the conversions of some neutrino $\nu$ into a different type of neutrino, $\nu'$, occurs when one of these neutrinos is free-streaming while the other one is not. The free-streaming component is out of equilibrium with the rest of the star, which prevents the wash-out of the asymmetry. Second, the position of the transition point it affected by the magnetic field. I will review two possible explanations, which do not require any exotic neutrino interactions and rely only on the established neutrino properties, namely matter-enhanced neutrino oscillations. The additional assumptions about the existence of sterile neutrinos [15] and the neutrino masses appear plausible from the point of view of particle physics.

## II. PULSAR KICKS FROM NEUTRINO OSCILLATIONS

As neutrinos pass through matter, they experience an effective potential

$$V(\nu_s) = 0 \tag{1}$$

$$V(\nu_e) = -V(\bar{\nu}_e) = V_0 \,(3\, Y_e - 1 + 4\, Y_{\nu_e}) \tag{2}$$

$$V(\nu_{\mu,\tau}) = -V(\bar{\nu}_{\mu,\tau}) = V_0\,(Y_e - 1 + 2\,Y_{\nu_e}) \;+ \frac{eG_F}{\sqrt{2}} \left(\frac{3N_e}{\pi^4}\right)^{1/3} \frac{\vec{k}\cdot\vec{B}}{|\vec{k}|} \tag{3}$$

where $Y_e$ ($Y_{\nu_e}$) is the ratio of the number density of electrons (neutrinos) to that of neutrons, $\vec{B}$ is the magnetic field, $\vec{k}$ is the neutrino momentum, $V_0 = 10\,\mathrm{eV}\,(\rho/10^{14}\mathrm{g\,cm}^{-3})$. The magnetic field dependent term in equation (3) arises from a one-loop finite-density contribution [17,18] to the self-energy of a neutrino propagating in a magnetized medium. An excellent review of the neutrino "refraction" in magnetized medium is found in Ref. [18].

The condition for resonant [19] oscillation $\nu_i \leftrightarrow \nu_j$ is

$$\frac{m_i^2}{2k}\,cos\,2\theta_{ij} + V(\nu_i) = \frac{m_j^2}{2k}\,cos\,2\theta_{ij} + V(\nu_j) \tag{4}$$

where $\nu_{i,j}$ can be either a neutrino or an anti-neutrino.

The neutron star can receive a kick if the following two conditions [11–13] are satisfied: (1) the adiabatic[1] oscillation $\nu_i \leftrightarrow \nu_j$ occurs at a point inside the $i$-neutrinosphere but outside the $j$-neutrinosphere; and (2) the difference $[V(\nu_i) - V(\nu_j)]$ contains a piece that depends on the relative orientation of the magnetic field $\vec{B}$ and the momentum of the outgoing neutrinos, $\vec{k}$. If the first condition is satisfied, the effective neutrinosphere of $\nu_j$ coincides with the surface formed by the points of resonance. The second condition ensures that this surface (a "resonance-sphere") is deformed by the magnetic field in such a way that it will be further from the center of the star when $(\vec{k}\cdot\vec{B}) > 0$, and nearer when $(\vec{k}\cdot\vec{B}) < 0$. The average momentum carried away by the neutrinos depends on the temperature of the region from which they exit. The deeper inside the star, the higher is the temperature during the neutrino cooling phase. Therefore, neutrinos coming out in different directions carry momenta which depend on the relative orientation of $\vec{k}$ and $\vec{B}$. This causes the asymmetry in the momentum distribution. An 1% asymmetry is sufficient to generate birth velocities of pulsars consistent with observation.

Let us use two different models for the neutrino emission to calculate the kick from the active-sterile and the active neutrinos, respectively. As shown in Ref. [13], these two models are in good agreement.

## III. OSCILLATIONS INTO STERILE NEUTRINOS

Since the sterile neutrinos have a zero-radius neutrinosphere, $\nu_s \leftrightarrow \bar{\nu}_{\mu,\tau}$ oscillations can be the cause of the pulsar motions if $m(\nu_s) > m(\nu_{\mu,\tau})$. If, on the other hand, $m(\nu_s) < m(\nu_{\mu,\tau})$, $\nu_s \leftrightarrow \nu_{\mu,\tau}$ oscillations can play the same role.

In the presence of the magnetic field, the condition (4) is satisfied at different distances $r$ from the center (Fig. 1), depending on the value of the $(\vec{k}\cdot\vec{B})$ term in (4). The surface of the resonance is, therefore,

$$r(\phi) = r_0 + \delta\;cos\,\phi, \tag{5}$$

where $cos\,\phi = (\vec{k}\cdot\vec{B})/k$ and $\delta$ is determined by the equation $(dN_n(r)/dr)\delta \approx e\left(3N_e/\pi^4\right)^{1/3} B$. This yields [12]

$$\delta = \frac{e\mu_e}{\pi^2}\,B \Bigg/ \frac{dN_n(r)}{dr}\;, \tag{6}$$

where $\mu_e \approx (3\pi^2 N_e)^{1/3}$ is the chemical potential of the degenerate (relativistic) electron gas.

Assuming a black-body radiation luminosity $\propto T^4$ for the effective neutrinosphere, the asymmetry in momentum distribution [12] is

$$\frac{\Delta k}{k} = \frac{4e}{3\pi^2}\left(\frac{\mu_e}{T}\frac{dT}{dN_n}\right) B, \tag{7}$$

[1] Non-adiabatic oscillations are discussed in Ref. [16]

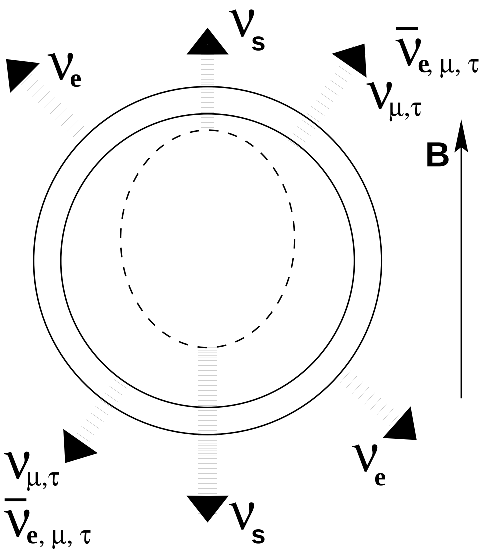


FIG. 1. Sterile neutrinos produced by oscillations are emitted from regions of different temperatures inside a neutron star.

To calculate the derivative in (7), we use the relation between the density and the temperature of a non-relativistic Fermi gas. Finally,

$$\frac{\Delta k}{k} = \frac{4e\sqrt{2}}{\pi^2} \frac{\mu_e \mu_n^{1/2}}{m_n^{3/2} T^2} B = 0.01 \frac{B}{3 \times 10^{15} \mathrm{G}} \tag{8}$$

if the neutrino oscillations take place in the core of the neutron star, at density of order $10^{14}\,\mathrm{g\,cm^{-3}}$. The neutrino oscillations take place at such a high density if one of the neutrinos has mass in the keV range, while the other one is much lighter.

The mixing angle can be very small, because the adiabaticity condition is satisfied if

$$l_{osc} \approx \left( \frac{1}{2\pi} \frac{\Delta m^2}{2k} \, sin\, 2\theta \right)^{-1} \approx \frac{10^{-2}\,\mathrm{cm}}{sin\, 2\theta} \tag{9}$$

is smaller than the typical scale of the density variations. Thus the oscillations remain adiabatic as long as $sin^2\, 2\theta > 10^{-8}$.

## IV. OSCILLATIONS OF ACTIVE NEUTRINOS

The active neutrino oscillations can also explain the pulsar kick [11]. The magnitude of the kick can be calculated using a model for neutrino transfer used in the previous section [11]. That is, one can assume that the neutrinos are emitted from a "hard" neutrinosphere with temperature $T(r)$ and that their energies are described by the Stefan-Boltzmann law. Alternatively, we can use the Eddington model for the atmosphere which was used by Schinder and Shapiro [21] to describe the emission of a single neutrino species. One can generalize it to include several types of neutrinos.[2]

In the diffusion approximation, the distribution functions $f$ are taken in the form [21]:

[2] A recent attempt [20] to use the Eddington model for the neutrino transfer failed to produce a correct result because the neutrino absorption $\nu_e n \to e^- p^+$ was neglected, and also because the different neutrino opacities were assumed to be equal to each other. The assumption [20] that the effect of neutrino oscillations can be accounted for in a simplistic model with one neutrino species and a deformed core-atmosphere boundary is also incorrect because the temperature profile is determined by the emission of six neutrino types, five of which are emitted isotropically. The neutrinos of the sixth flavor, which have an

$$f_{\nu_i} \approx f_{\bar{\nu}_i} \approx f^{eq} + \frac{\xi}{\Lambda_i} \frac{\partial f^{eq}}{\partial m}, \tag{10}$$

where $f^{eq}$ is the distribution function in equilibrium, $\Lambda_i$ denote the respective opacities, $m$ is the column mass density, $m = \int \rho \, dx$, $\xi = cos\alpha$, and $\alpha$ is the normal angle of the neutrino velocity to the surface. At the surface, one imposes the same boundary condition for all the distribution functions, namely

$$f_{\nu_i}(m, \xi) = \begin{cases} 0, & \text{for } \xi < 0, \\ 2f^{eq}, & \text{for } \xi > 0. \end{cases} \tag{11}$$

However, the differences in $\Lambda_i$ produce the unequal distributions for different neutrino types.

Generalizing the discussion of Refs. [21] to include six flavors, three neutrinos and three antineutrinos, one can write the energy flux as

$$F = 2\pi \int_0^\infty E^3 dE \int_{-1}^{1} \xi d\xi \, \sum_{i=1}^{3} (f_{\nu_i} + f_{\bar{\nu}_i}), \tag{12}$$

We will assume that $\Lambda_i = \Lambda_i^{(0)} (E^2/E_0^2)$.

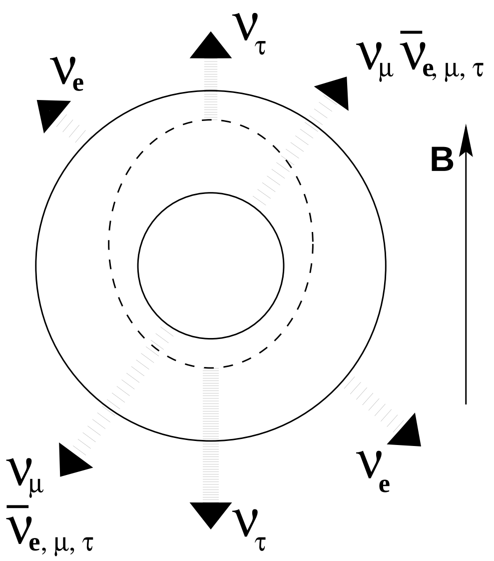


FIG. 2. Oscillations $\nu_\tau \leftrightarrow \nu_e$ result in anisotropic emission of $\tau$-neutrinos. The upward-going $\nu_\tau$'s escape from the region with a lower temperature than that of the downward-going $\tau$-neutrinos.

We use the expressions for $f_{\nu_i}$ from equation (10). Changing the order of differentiation with respect to $m$ and integration over $E$ and $\xi$, and using the fact that $f^{eq}$ is isotropic, we arrive at the result similar to that of Ref. [21]:

$$F = \frac{2\pi^3}{9} E_0^2 \left[ \sum_{i=1}^{3} \frac{2}{\Lambda_i^{(0)}} \right] \frac{\partial T^2}{\partial m}. \tag{13}$$

The basic assumption of the model is that flux $F$ is conserved. In other words, the neutrino absorptions $\nu_e n \rightarrow e^- p^+$ are neglected. Since the sum in brackets, as well as the flux $F$ are treated [21] as constants with respect to $m$, one can solve for $T^2$:

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anisotropic momentum distribution, cause negligible (down by at least a factor of 6) asymmetry in the temperature profile. When the neutrino absorption is included, the Eddington model gives the same result for the kick [13] as the model with "hard neutrinospheres" [11,12].

$$T^2(m) = \frac{9}{2\pi^3} E_0^{-2} \left[ \sum_{i=1}^{3} \frac{2}{\Lambda_i^{(0)}} \right]^{-1} F\, m + \left( \frac{30}{7\pi^5} F \right)^{1/2} \tag{14}$$

Swapping the two flavors in equation (14) leaves the temperature unchanged in the Eddington approximation. Hence, neutrino oscillations do not alter the temperature profile in this approximation.

We will now include the absorptions of neutrinos.

Some of the electron neutrinos are absorbed on their passage through the atmosphere thanks to the charged-current process

$$\nu_e n \rightarrow e^- p^+. \tag{15}$$

The cross section for this reaction is $\sigma = 1.8\, G_F^2 E_\nu^2$, where $E_\nu$ is the neutrino energy. The total momentum transfered to the neutron star by the passing neutrinos depends on the energy.

Both numerical and analytical calculations show that the muon and tau neutrinos leaving the core have much higher mean energies than the electron neutrinos [22,23]. Below the point of MSW [19] resonance the electron neutrinos have the mean energies $\approx 10$ MeV, while the muon and tau neutrinos have energies $\approx 25$ MeV.

The origin of the kick in this description is that the neutrinos spend more time as energetic electron neutrinos on one side of the star than on the other side, hence creating the asymmetry. Although the temperature profile remains unchanged in Eddington approximation, the unequal numbers of neutrino absorptions push the star, so that the total momentum is conserved.

Below the resonance $E_{\nu_e} < E_{\nu_{\tau,\mu}}$. Above the resonance, this relation is inverted. The energy deposition into the nuclear matter depends on the distance the electron neutrino has traveled with a higher energy. This distance is affected by the direction of the magnetic field relative to the neutrino momentum.

We assume that the resonant conversion $\nu_e \leftrightarrow \nu_\tau$ takes place at the point $r = r_0 + \delta(\phi); \delta(\phi) = \delta_0 \cos\phi$. The position of the resonance depends on the magnetic field $B$ inside the star [11]:

$$\delta_0 = \frac{e \mu_e B}{2\pi^2} \Big/ \frac{dN_e}{dr}\ , \tag{16}$$

where $N_e \approx Y_e N_n$ is the electron density and $\mu_e$ is the electron chemical potential.

Below the resonance the $\tau$ neutrinos are more energetic than the electron neutrinos. The oscillations exchange the neutrino flavors, so that above the resonance the electron neutrinos are more energetic than the $\tau$ neutrinos. The number of neutrino absorptions in the layer of thickness $2\delta(\phi)$ around $r_0$ depends on the angle $\phi$ between the neutrino momentum and the direction of the magnetic field. Each occurrence of the neutrino absorption transfers the momentum $E_{\nu_e}$ to the star. The difference in the numbers of collisions per electron neutrino between the directions $\phi$ and $\pi + \phi$ is

$$\Delta k_e / E_{\nu_e} = 2\, \delta(\phi)\, N_n\, [\sigma(E_1) - \sigma(E_2)] \tag{17}$$

$$= 1.8\, G_F^2 [E_1^2 - E_2^2]\, \frac{\mu_e}{Y_e} \frac{eB}{\pi^2}\, h_{N_e}\, \cos\phi, \tag{18}$$

where $h_{N_e} = [d(\ln N_e)/dr]^{-1}$.

We use $Y_e \approx 0.1$, $E_1 \approx 25$ MeV, $E_2 \approx 10$ MeV, $\mu_e \approx 50$ MeV, and $h_{N_e} \approx 6$ km. After integrating over angles and taking into account that only one neutrino species undergoes the conversion, we obtain the final result for the asymmetry in the momentum deposited by the neutrinos:

$$\frac{\Delta k}{k} = 0.01 \frac{B}{2 \times 10^{14}\mathrm{G}}, \tag{19}$$

which agrees with the estimates[3] [11,24] that use a different model for the neutrino emission.

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[3]We note in passing that we estimated the kick in Refs. [11,12] assuming $\mu_e \approx$ const. A different approximation, $Y_e \approx$ const, gives a somewhat higher prediction for the magnitude of the magnetic field [24].

Neutrinos also lose energy by scattering off the electrons. Since the electrons are degenerate, the final-state electron must have energy greater than $\mu_e$. Therefore, electron neutrinos lose from 0.2 to 0.5 of their energy per collision in the neutrino-electron scattering. However, since $N_e \ll N_n$, this process can be neglected.

One may worry whether the asymmetric absorption can produce some back-reaction and change the temperature distribution inside the star altering our result (19). If such effect exists, it is beyond the scope of Eddington approximation, as is clear from equation (14). The only effect of the asymmetric absorption is to make the star itself move, in accordance with the momentum conservation. This is the origin of the kick (19).

Of course, in reality the back-reaction is not exactly zero. The most serious drawback of Eddington model, pointed out in Ref. [21], is that diffusion approximation breaks down in the region of interest, where the neutrinos are weakly interacting. Another problem has to do with inclusion of neutrino absorptions and neutrino oscillations [21]. However, to the extent we believe this approximation, the pulsar kick is given by equation (19).

## V. CONCLUSION

The neutrino oscillations can explain the motions of pulsars. Although many alternatives have been proposed, all of them fail to explain the large magnitudes of the pulsar velocities.

If the pulsar kick velocities are due to $\nu_e \leftrightarrow \nu_{\mu,\tau}$ conversions, one of the neutrinos must have mass $\sim$ 100 eV (assuming small mixing) and must decay on the cosmological time scales not to overclose the Universe [11]. This has profound implications for particle physics hinting at the existence of Majorons [25] or other physics beyond the Standard Model that can facilitate the neutrino decay.

If the active-to-sterile neutrino oscillations [12] are responsible for pulsar velocities, the prediction for the sterile neutrino to have a mass of several keV is not in contradiction with any of the present bounds. In fact, the $\sim$keV mass sterile neutrino has been proposed as a dark-matter candidate [26].

Some other explanations [14] that utilize new hypothetical neutrino properties, but use a similar mechanism for generating the asymmetry, can also explain large pulsar velocities.

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