What are sterile neutrinos good for?

Mitesh Patel and George M. Fuller
Department of Physics, University of California, San Diego, La Jolla, CA 92093-0350

Taken at face value, current experimental data indicate the existence of a new particle, the sterile neutrino, which must be a singlet under the Standard Model gauge group. Although they are not detectable through traditional means, such particles have interesting *observable* consequences for particle astrophysics and cosmology. Here we examine these implications and discuss, in particular, the relationship between matter-enhanced active-sterile neutrino transformation and the synthesis of heavy elements in supernovae.

I. INTRODUCTION

In 1930, after inferring the existence of the neutrino from the continuous electron spectrum of nuclear β-decay, W. Pauli remarked [1], "I have done a terrible thing. I have postulated a particle that cannot be detected." Twenty-three years later, F. Reines and C. L. Cowan [2] reported the first detection of neutrinos via inverse β-decay.

Encouraged by these events, the propensity of history to repeat itself, and a steady stream of positive experimental data, contemporary particle physicists and astrophysicists have recently explored in detail the ramifications of so-called "sterile" neutrinos, generically denoted ν_s. The existence of such Standard Model-singlet fermions, which couple to the conventional (or "active") left-handed neutrinos ν_L solely through effective mass terms, is implied by the confluence of several neutrino oscillation experiments:

- **Atmospheric neutrinos** The Super-Kamiokande Collaboration has reported convincing evidence for the suppression of the flux of ν_μ and $\bar{\nu}_\mu$ produced in the Earth's upper atmosphere by cosmic rays [3]. (The measured flux of ν_e and $\bar{\nu}_e$ is within expectation.) In particular, there is a statistically significant zenith angle dependence for the high energy muon-like events which is fit well by neutrino oscillations.[1] A two-neutrino vacuum mixing fit yields $\delta m^2 \approx 10^{-3}$ eV^2 and $\sin^2(2\theta) \approx 1$, if the neutrino mixing maximally with ν_μ is ν_τ [3]. There are also matter-enhanced (Mikheyev-Smirnov-Wolfenstein or MSW [5]) solutions $\delta m^2 = \pm 5 \times 10^{-3}$ eV^2 and $\sin^2(2\theta) \approx 1$ if the mixing partner is ν_s [6].
- **Solar neutrinos** An array of solar neutrino experiments [7] has observed an energy-dependent deficit of ν_e emitted by fusion processes in the sun. The Kamiokande and Super-Kamiokande experiments observe about one-half of the expected flux of the highest energy solar neutrinos. The Homestake chlorine experiment, sensitive to intermediate and higher energy neutrinos, sees approximately one-third of the expected flux. Further, the SAGE and Gallex gallium experiments record roughly one-half of the expected flux integrated over nearly the entire solar spectrum. The combined result is a distorted spectrum which is extremely difficult to account for by solely astrophysical means. Global two-neutrino fits to these observations yield [7] the large[2] ($\delta m^2 \approx 10^{-5}$ eV^2, $\sin^2(2\theta) \approx 1$) and small ($\delta m^2 \approx 10^{-5}$ eV^2, $\sin^2(2\theta) \approx 5 \times 10^{-3}$) angle MSW solutions and the "just-so" vacuum ($\delta m^2 \approx 10^{-10}$ eV^2, $\sin^2(2\theta) \approx 1$) solution. The neutrino mixing with ν_e may be ν_μ, ν_τ, or ν_s, depending on the solution.[3]
- **Accelerator neutrinos** The Los Alamos LSND experiment has recorded an excess of ν_e and $\bar{\nu}_e$ events in accelerator-produced beams of ν_μ and $\bar{\nu}_\mu$ respectively [8]. There are several allowed regions in the oscillation parameter space, all of which fall in the ranges 0.2 eV$^2 \lesssim \delta m^2 \lesssim 8$ eV^2 and $10^{-3} \lesssim \sin^2(2\theta) \lesssim 10^{-1}$, assuming

---

[1] Continued observations and future long base-line accelerator experiments will help rule out other explanations for the anomaly. See Ref. [4] for a catalog and discussion of these "non-standard" solutions.

[2] Including all available data in the fit removes the large-angle MSW solution. See Ref. [7] for details.

[3] Of course, requiring compatibility with other experiments restricts the allowed oscillation channels.

two-neutrino mixing. The agreement [8] between the regions for the neutrino ($\nu_\mu \to \nu_e$) and antineutrino ($\bar{\nu}_\mu \to \bar{\nu}_e$) channels reinforces the oscillation interpretation. The KARMEN experiment has searched for excess events in the same channels, but it is not yet clear whether its results contradict the positive LSND signal.

The mutual incompatibility of these disparate sets of results is *prima facie* evidence for the existence of a *light* sterile neutrino ν_s, since the number of light weakly interacting neutrino species is known to be three (namely, ν_e, ν_μ, and ν_τ) [9], an effectively three-neutrino mass matrix yields at most two independent δm^2's, and the active neutrinos are known to be very light compared to their charged counterparts.[4] Global fits of the data to three-neutrino mass matrices have borne out this conclusion, and several authors have indicated how to accommodate all of the data in a four-neutrino mixing matrix [10].

Assuming that the current data is explained by oscillations and that future neutrino experiments compel us to confront the reality of an additional light neutrino species, a number of workers have begun constructing theoretical models which yield the naturally small Dirac *and* Majorana neutrino masses required for appreciable active-sterile neutrino mixing. Some of these models are generalizations of the traditional see-saw mechanism for generating light active neutrinos in the presence of very heavy sterile neutrinos [11]. Many methods rely on new symmetries to ensure that both active and sterile neutrino masses are small and comparable. All of them can be classified as simple extensions of the Standard Model (SM) gauge group and matter content (including Grand Unified Theories or GUTs), supersymmetric models, or superstring inspired scenarios.

The phenomenological consequences and uses of sterile neutrinos are equally interesting. In particular, resonant transitions among active and sterile neutrinos can alter significantly the dynamics of early universe cosmology and various astrophysical venues. Indeed, significant consequences are almost guaranteed in phenomena such as Big Bang nucleosynthesis (BBN) or core-collapse supernovae, whose outcome is determined or dominated by neutrino physics. Transitions to and from sterile neutrinos can distort severely the active neutrinos' energy spectra, resulting, for example, in nucleosynthetic abundances markedly different from the commonly accepted values.

These effects are not invariably unfavorable, however. Indeed, sterile neutrinos have been variously invoked to explain the origin of pulsar kicks [12], provide a new dark matter candidate [13], account for the diffuse ionization in the Milky Way galaxy [14], resolve the "crisis" in BBN [15], and help enable the synthesis of heavy elements in Type II supernovae [16,17]. In these proceedings, we describe the interesting phenomenological implications of two of these scenarios. We recapitulate in Sec. II the physics of sterile neutrino dark matter, concentrating on the production of the cold, non-thermal variety. In Sec. III, we summarize our recent work on and the status of matter-enhanced active-sterile neutrino transformation solutions to heavy-element nucleosynthesis. We give conclusions in Sec. IV.

II. STERILE NEUTRINO DARK MATTER

The number of suitable candidates for the dark matter of the universe is staggering! In Eq. (1) we have listed some of the possible constituents — without regard to their potential mutual exclusivity — of the total mass-energy of the universe (measured as a fraction Ω of the Friedmann-Robertson-Walker (FRW) closure energy density):[5]

$$\begin{aligned}\Omega_{\rm TOTAL} = {} & \Omega_{\rm baryon} + \Omega_\Lambda + \Omega_{\rm axion} + \Omega_{\nu_{e,\mu,\tau}} + \Omega_{\rm Q-ball} \\ & + \Omega_{\rm LSP} + \Omega_{\rm WIMP} + \Omega_{\rm WIMPZILLA} + \Omega_{\rm SUSY} + \Omega_{\rm CMBR} \\ & + \Omega_{\rm monopole} + \Omega_{\rm starlight} + \Omega_{\rm cosmic\ string} + \Omega_{\rm primordial\ black\ hole}\end{aligned}$$

---

[4] If the additional neutrino is not a SM singlet, then it must have *very weak* interactions with the SM particles in order to meet these constraints.

[5] A number of these dark matter hopefuls have been culled from Ref. [18] and preprint archive listings [19]. See Ref. [18] for a lucid discussion of the dark matter problem and structure formation.

$$\begin{aligned}&+ \Omega_{\tilde{\chi}^0} + \Omega_{\text{majoron}} + \Omega_{\text{moron}} + \Omega_{\tilde{\gamma}} + \Omega_{\text{Newtorite}} + \Omega_{\text{quark nugget}}\\ &+ \Omega_{\tilde{\nu}} + \Omega_{\text{pyrgon}} + \Omega_{\tilde{\mathrm{g}}} + \Omega_{\text{gravitational wave}} + \Omega_{\text{maximon}}\\ &+ \Omega_{\tilde{\mathrm{h}}} + \Omega_{\text{familon}} + \Omega_{\text{tetron}} + \Omega_{\text{penton}} + \Omega_{\text{hexon}} + \Omega_{\text{crypton}}\\ &+ \boldsymbol{\Omega}_{\boldsymbol{\nu}_{\mathbf{s}}} \end{aligned} \tag{1}$$

The astute reader will have noticed the addition of sterile neutrino dark matter to this list. If one light sterile species ν_s is required by the neutrino oscillation experiments, then the multi-generational structure of the SM argues strongly in favor of two additional steriles ν_s' and ν_s''. While more massive than their lighter sibling, these additional neutrinos could also have relatively small masses which are easily compatible with the data.

If there is a ν_s' in the 200 eV to 10 keV mass range and a primordial lepton number $L \approx 10^{-3} - 10^{-1}$ in any of the active neutrino flavors, Shi & Fuller [13] have shown that this asymmetry can drive resonant production of sterile neutrino dark matter in the early universe. Furthermore, since the MSW mechanism here favors efficient conversion from active to sterile neutrinos only for the lowest energy neutrinos and the process itself destroys lepton number, the resulting ν_s' spectrum is non-thermal and essentially "cold."

Unlike conventional neutrino dark matter (Hot Dark Matter or HDM), this Cold Dark Matter (CDM) candidate has a relatively short free-streaming length at the epoch of structure formation, so density fluctuations may grow unimpeded on galactic scales. For example, a $\approx$ 500 eV sterile neutrino can contribute a fraction $\Omega_{\nu_s} \approx 0.4$ to the critical density and stream freely over only $\approx$ 0.4 Mpc, which corresponds to a structure cut-off $\approx 10^{10} M_\odot$, about the size of dwarf galaxies. For further details, please consult Ref. [13].

III. ACTIVE-STERILE NEUTRINO TRANSFORMATION AND HEAVY-ELEMENT NUCLEOSYNTHESIS

What is the origin of the heavy elements? The light elements (H, He, D, and Li) are produced in the early universe and the intermediate mass elements (up to Fe) during the "main sequence" evolution of stars,[6] but the source of at least half of the nuclides heavier than iron is unknown.

It is not difficult to understand why ordinary stellar evolution cannot produce nuclei more massive than iron. The thermonuclear processes which take place in stellar interiors work gradually from hydrogen toward iron by fusing together ever heavier nuclei. Since these thermal reactions can proceed only if the products are more tightly bound than the reactants, the path of stellar nucleosynthesis ends at iron, which is the most tightly bound nucleus.

It is clear, then, that elements heavier than iron must form via some other process(es) in some other environment(s). One pathway from iron to heavy nuclides such as plutonium, iodine, tin, lead, and gold is the **r**-process, so named because it proceeds via the **r**apid capture of neutrons on iron-sized "seed" nuclei. In the r-process, seed nuclei capture neutrons so rapidly that they can leap past iron on the curve of binding energy versus atomic mass. After the r-process is complete, the now extremely neutron-rich nuclei β-decay to more stable elements on the periodic table, yielding some mass distribution of nuclides.[7] A key necessary condition for a successful r-process is a preponderance of neutrons over protons, as the heavy elements are quite neutron-rich.[8] Once seed nuclei are available for neutron capture to proceed, this requirement becomes having a large number of free neutrons for each seed nucleus.

Although the physics of the r-process is relatively well-known, the astrophysical environment in which it takes place has not been determined conclusively. The major contenders include Type II (or core-collapse) supernovae and binary neutron star mergers. The former is currently the most promising r-process site, and we now restrict ourselves to a

---

[6] See Ref. [20] for a detailed exposition on stellar structure and dynamics.

[7] Preferably a distribution which matches the observed abundances!

[8] Neutron-richness of the heavy nuclides can be understood roughly as a compromise among Pauli's exclusion principle, long-range Coulomb forces between protons, and short-range nuclear (strong) forces between nucleons.

discussion of supernova physics and how non-standard particle physics like neutrino oscillations can help enable the r-process.

A Type II supernova results from the catastrophic gravitational collapse of a massive star [20,21]. Specifically, a star with mass $M \gtrsim 8M_\odot$ burns successively heavier fuels throughout its life, eventually accumulating an inner iron core. As indicated above, thermal fusion processes cannot extract energy from iron to provide pressure support against gravity, so the core eventually collapses under its own gravity. Abetted by the photodissociation of the iron nuclei, the core rapidly neutronizes via electron capture on protons. When the central core density reaches the saturation density of nuclear matter, the nucleons touch, and the core "bounces," yielding an outgoing shockwave. Unfortunately, the shock wave eventually dies out, as it loses all of its energy dissociating the mantle of the star. Meanwhile, however, thermally produced neutrinos, which were originally trapped in the dense core, escape and revive the shock (the supernova explodes!). The remaining neutrinos continue to leak out on a diffusion time scale and for 10-20 seconds drive significant mass loss from the newly formed proto-neutron star.

It is in this neutrino-driven "wind" [22] that the r-process is believed to occur [23]. The hot proto-neutron star emits neutrinos of all flavors, with average energies $\langle E_{\nu_\delta}\rangle \approx \langle E_{\bar{\nu}_\delta}\rangle \gtrsim \langle E_{\bar{\nu}_e}\rangle \gtrsim \langle E_{\nu_e}\rangle$, where $\delta = \mu, \tau$ refers to the muon and tau neutrinos, and comparable luminosities. They stream nearly freely through a plasma of electrons, positrons, neutrons, and protons. By exchanging energy with the cooler plasma, the neutrinos drive the r-process ingredients out of the gravitational potential well of the neutron star. As a fluid element of this neutrino-heated ejecta travels away from the surface of the star, its temperature gradually drops, its velocity increases, and the following sequence ensues:

- The reactions

$$\nu_e + n \longleftrightarrow p + e^- \tag{2}$$

$$\bar{\nu}_e + p \longleftrightarrow n + e^+ \tag{3}$$

  set the ratio n/p of the number densities of neutrons and protons.[9] This number is usually written in terms of the electron fraction Y_e, the net number of electrons per baryon. Charge neutrality of the plasma gives $Y_e = 1/(1 + n/p)$. Thus a higher (lower) n/p corresponds to a lower (higher) Y_e. The reactions in Eqs. (2-3) freeze out at a temperature $T \approx 0.8$ MeV, when their rates fall below the local expansion rate.

- When the plasma cools to $T \approx 0.75$ MeV, α-particles (^{4}He nuclei) start to form from the ambient neutrons and protons.

- At lower plasma temperatures, some of the α-particles combine via three-body interactions into seed nuclei with mass number $A = 50 - 100$.

- Finally, at sufficiently low temperatures ($T \approx 0.25$ MeV), the r-process occurs.

Unfortunately, detailed numerical simulations [24] have found that the r-process is *precluded* in neutrino-heated ejecta by a phenomenon termed the "α-effect." The α-effect occurs when nucleons are taken up into α-particles, leaving nearly inert ^{4}He nuclei and excess neutrons (the plasma is already neutron-rich). Since the ν_e flux is still sizable in the region beyond freeze-out, the forward reaction in Eq. (2) proceeds to convert the leftover neutrons into protons. These protons and remaining neutrons quickly form additional α-particles before the forward reaction in Eq. (3) can reconvert the newly-formed protons. Ultimately, nearly all of the free neutrons end up in ^{4}He nuclei, the electron fraction is driven to 0.5, and the final neutron-to-seed ratio is too small to give even an anemic r-process!

---

[9] The reactions $n \longleftrightarrow p + e^- + \bar{\nu}_e$ contribute only negligibly to n/p for the relevant neutrino energies.

We have shown in Fig. 1 the radial evolution of the electron fraction in the hot, high-entropy "bubble" above the proto-neutron star. We have used a simple model [24] for the wind and taken representative values of the neutron star radius ($R = 10$ km), expansion time scale ($\tau = 0.3$ s), entropy per baryon ($s/k_B = 100$), and neutrino spectral parameters [17,16]. The α-effect begins when the temperature in the plasma drops to ≈ 0.75 MeV. Comparison of the behavior with and without the α-effect illustrates its negative impact.

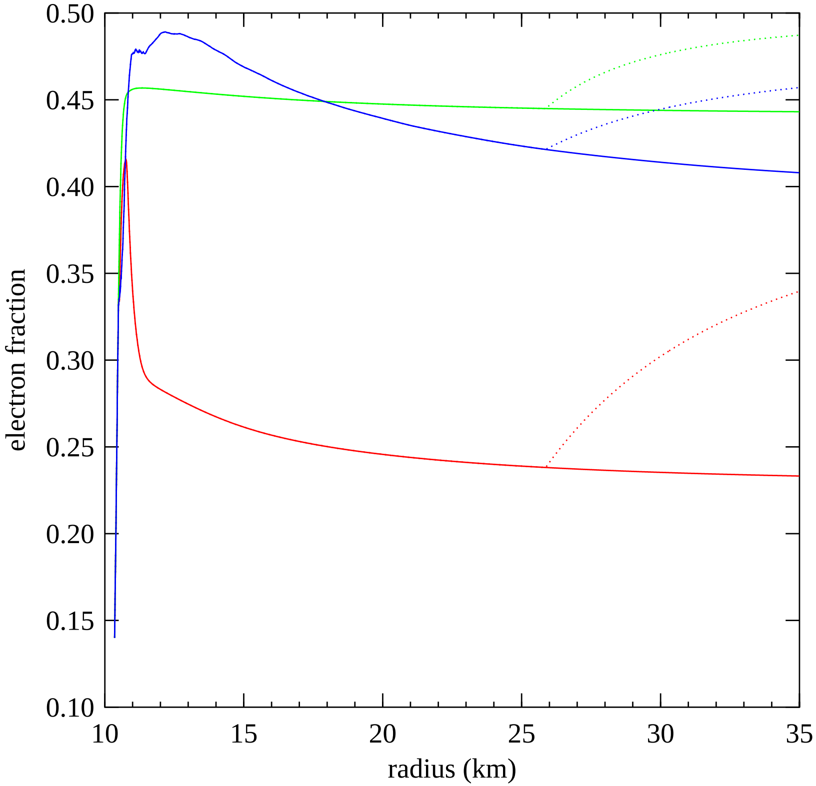

FIG. 1. Evolution of the electron fraction Y_e with radius in the neutrino-driven wind above a proto-neutron star for no neutrino oscillations (green), $\nu_e \leftrightarrow \nu_s$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_s$ oscillations ($\delta m^2 = 10$ eV^2, $\sin^2(2\theta) = 0.01$) without neutrino background effects (red), and such oscillations with neutrino background effects (blue). The solid curves indicate the behavior without the α-effect and the dotted curves the behaviour with the α-effect. See the text for the relevant physical parameters.

Various calculations [24,16,17] have shown that this problem persists for all reasonable variations in wind outflow and neutrino spectral parameters. Since there is mounting evidence [24,27,25,26] that the r-process must occur in neutrino-heated ejecta in Type II supernovae, a number of authors [16,17,28] have attempted to circumvent the α-effect in order to enable heavy-element nucleosynthesis.

The high neutron-to-seed ratio requirement for a successful r-process is met if Y_e is sufficiently small ($Y_e < 1/2$ corresponds to neutron-rich ejecta); the expansion rate of the ejecta is sufficiently high that the three-body reactions making seed nuclei are inefficient; and/or the entropy of the plasma is high enough that nucleons prefer to remain free rather than bound in nuclei. Increasing the expansion rate will not redress the α-effect, since α-particles form via relatively fast two-body processes. Raising the entropy in the ejecta substitutes for the α-effect another process which decimates the neutron-to-seed ratio: neutrino neutral current spallation of ^{4}He nuclei [29]. Since the severity of the effect depends on the ν_e flux in the region of α-particle production, lowering Y_e without changing the ν_e flux also cannot help enable the r-process.

Non-standard neutrino physics such as neutrino oscillations is an elegant solution to this problem, for resonant transformation between ν_e and some other species can directly influence the ν_e flux. Given the hierarchy of the energies of the active neutrinos, matter-enhanced transformation between ν_e and ν_μ or ν_τ will actually enhance the α-effect. Transformations among the active antineutrinos will affect only $\bar{\nu}_e$. As discussed in the Introduction, however, various neutrino experiments imply the existence of a sterile neutrino which mixes appreciably with the active neutrinos. As long as the neutron star emits a negligible number of sterile neutrinos, effective active-sterile mixing in the form of $\nu_e \leftrightarrow \nu_s$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_s$ has the potential to forestall the α-effect by removing the offending ν_e's [16,17,28].

The authors of Ref. [16] and we [17] have recently investigated this mixing scheme in neutrino-heated ejecta. Shown in Fig. 1 is the evolution of Y_e with radius for a representative set of mixing parameters. In Ref. [16], the authors included all contributions to the effective mass [30] of electron neutrinos except neutrino forward scattering on the "background" of other neutrinos emitted by the neutron star [31]. In Ref. [17], we have also included this many-body effect. The behavior of Y_e for both analyses is indicated in the figure, both with and without the α-effect. In Fig. 2, we have shown final electron fraction attained (at $r = 35$ km in Fig. 1) without neutrino background effects.

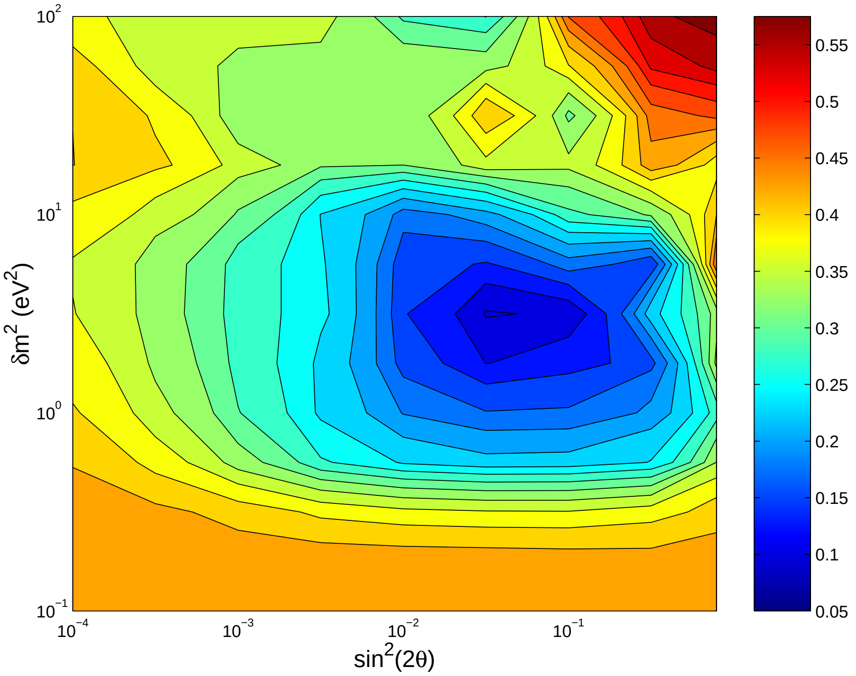

FIG. 2. Final electron fraction Y_e, excluding neutrino background effects, for various $\nu_e \leftrightarrow \nu_s$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_s$ oscillation parameters δm^2 and $\sin^2(2\theta)$. The color bar at the right indicates the value of Y_e. The physical conditions are the same as in Fig. 1.

In the absence of the neutrino background, active-sterile mixing is a viable solution to the r-process problem. For a relatively large range of neutrino mixing parameters, the final electron fraction is less than 0.3, yielding a highly neutron-rich r-process environment. Calculations with various choices of the outflow and neutrino spectral parameters have confirmed that this is also a *robust* solution [16,17,28]: a high neutron-to-seed ratio obtains for a wide range of supernova and mixing parameters. An additional benefit is that the δm^2-$\sin^2(2\theta)$ region favored for enabling the r-process coincides with one of the four-neutrino mixing schemes explaining the current experiments [16,10].

The presence of the neutrino background unfortunately mitigates this remedy, but the background gradually dies away as neutrinos diffuse out of the neutron star. At sufficiently late times after the supernova bounce and explosion, the probability of neutrino-neutrino forward scattering is sufficiently small that the scenario of Ref. [16] prevails. At these times, however, the neutrino fluxes have fallen sufficiently that there may not be a need for neutrino transformation to relieve the α-effect. If the total neutron-rich mass ejected at late times is sufficient to account for all of the r-process material in the galaxy, then there is no need to invoke non-standard neutrino physics. Settling this issue requires a self-consistent calculation of the wind coupled with neutrino oscillations and including neutrino background effects. On the other hand, if future experiments conclusively establish the existence of a light sterile neutrino in the mass range relevant for Type II supernovae, active-sterile mixing may have profound consequences for the synthesis of the heavy elements.

IV. CONCLUSION

The present solar, atmospheric, and accelerator neutrino experiments suggest the existence of a light sterile neutrino. While this is a radical departure from the folklore that sterile neutrinos (if they exist) are very heavy, future experiments [10] may confirm their reality and force theorists to modify or discard cherished models of neutrino mass. As we have indicated in this paper, the mixing of sterile and active neutrinos has potentially far-reaching consequences for cosmology and astrophysics. They may account for much of the dark matter of the universe. They may even be the reason why we have gold rings, tin cans, atomic bombs, lead shielding,....

MP is supported in part by the NASA Graduate Student Researchers Program. GMF is partially supported by NSF grant PHY98-00980.

---

[1] F. Reines, in *History of Original Ideas and Basic Discoveries in Particle Physics* (ed. H. B. Newman and T. Ypsilantis), Plenum Press, New York (1996).
[2] F. Reines and C. L. Cowan, *Phys. Rev.* **90**, 492 (1953).
[3] Y. Fukuda, *et al.*, *Phys. Rev. Lett.* **81**, 1562 (1998); *Phys. Lett.* **B436**, 33 (1998).
[4] S. Pakvasa, preprint hep-ph/9905426.
[5] L. Wolfenstein, *Phys. Rev.* **D17**, 2369 (1978); **D20**, 2634 (1979); S. P. Mikheyev and A. Yu. Smirnov, *Sov. Phys. JETP* **69**, 4 (1986).
[6] O. Yasuda, preprint hep-ph/9809206.
[7] J. Bahcall, P. I. Krastev, and A. Yu. Smirnov, *Phys. Rev.* **D58**, 096016 (1998), and references therein.
[8] C. Athanassopoulos, *et al.*, *Phys. Rev. Lett.* **75**, 2650 (1996); **77**, 3082 (1996); **81**, 1774 (1998); *Phys. Rev.* **C54**, 2685 (1996); **C58**, 2489 (1998).
[9] C. Caso, *et al.*, *Eur. Phys. J.* **C3**, 1 (1998).
[10] S. M. Bilenky, C. Giunti, and W. Grimus, preprint hep-ph/9812360, and references therein.
[11] R. N. Mohapatra, preprint hep-ph/9808236, and references therein.
[12] A. Kusenko, these proceedings, preprint astro-ph/9903167, and references therein.
[13] X. Shi and G. M. Fuller, *Phys. Rev. Lett* **82**, 2832 (1999).
[14] R. N. Mohapatra and D. W. Sciama, preprint hep-ph/9811446.
[15] K. Abazajian, these proceedings, preprint astro-ph/9904052, and references therein.
[16] G. C. McLaughlin, J. M. Fetter, A. B. Balantekin, and G. M. Fuller, *Phys. Rev.* **C59**, 2873 (1999).
[17] M. Patel and G. M. Fuller, in preparation.
[18] E. W. Kolb and M. S. Turner, *The Early Universe*, Addison-Wesley, Reading, Massachusetts (1994).
[19] http://xxx.lanl.gov
[20] D. D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis*, Univ. of Chicago Press, Chicago (1983).
[21] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects*, Wiley-Interscience, New York (1983).
[22] R. C. Duncan, S. L. Shapiro, and I. Wasserman, *Astrophys. J.* **309**, 141 (1986); Y.-Z. Qian and S. E. Woosley, *Astrophys. J.* **471**, 331 (1996).
[23] B. S. Meyer, *et al.*, *Astrophys. J.* **399**, 656 (1992); S. E. Woosley, *et al.*, *Astrophys. J.* **433**, 229 (1994).
[24] G. M. Fuller and B. S. Meyer, *Astrophys. J.* **453**, 792 (1995); G. C. McLaughlin, G. M. Fuller and J. R. Wilson, *Astrophys. J.* **472**, 440 (1996); B. S. Meyer, G. C. McLaughlin, and G. M. Fuller, *Phys. Rev.* **C58**, 3696 (1998).
[25] Y.-Z. Qian, W. C. Haxton, K. Langanke, and P. Vogel, *Phys. Rev.* **C55**, 1532 (1997); W. C. Haxton, K. Langanke, Y.-Z. Qian, and P. Vogel, *Phys. Rev. Lett.* **78**, 2694 (1997); Y.-Z. Qian, preprint nucl-th/9712080.
[26] Y.-Z. Qian, P. Vogel, and G. J. Wasserburg, *Astrophys. J.* **494**, 285 (1998).
[27] G. C. McLaughlin, and G. M. Fuller, *Astrophys. J.* **464**, L143 (1996).
[28] D. O. Caldwell, G. M. Fuller, and Y.-Z. Qian, in preparation.
[29] B. S. Meyer, *Astrophys. J.* **449**, L55 (1995).
[30] D. Nötzold and G. Raffelt, *Nuc. Phys.* **B307**, 924 (1988).
[31] Y.-Z. Qian, *et al.*, *Phys. Rev. Lett* **71**, 1965 (1993); Y.-Z. Qian and G. M. Fuller, *Phys. Rev.* **D51**, 1479 (1995).