Active-Sterile Neutrino Mixing in the Early Universe and Primordial Nucleosynthesis

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We investigate the effects of matter-enhanced (MSW) transformation of neutrinos in the Early Universe on the primordial helium yield (Y_p). We find that Y_p is affected much more by the MSW-induced alterations in the neutrino energy spectra than by the associated change in expansion rate. Specifically, the absence due to transformation of low energy electron neutrinos can significantly affect neutron-proton weak interconversion rates through the lifting of Fermi-blocking of neutron decay at low energies and through halting low-energy neutrino capture on neutrons. We find that the change of Y_p within a causal horizon is $-0.005 \leq \delta Y_p \leq 0.013$ for $m^2_{\nu_{\mu,\tau}} - m^2_{\nu_s} \leq 10^4\,\mathrm{eV}^2$ in the case of $\nu_{\mu,\tau}$-ν_s-ν_e mixing, with the lower limit at $m^2_{\nu_{\mu,\tau}} - m^2_{\nu_s} \approx 100\,\mathrm{eV}^2$, and $-0.002 \leq \delta Y_p \leq 0.020$ for $m^2_{\nu_e} - m^2_{\nu_s} \leq 1\,\mathrm{eV}^2$ in the case of ν_e-ν_s mixing.

I. INTRODUCTION

Big Bang Nucleosynthesis (BBN) remains one of the most successful probes of early times in the hot big bang cosmology. Standard BBN (SBBN) assumes the Standard Model proposition of only three massless light neutrinos, leaving the primordial element abundances characterized by only one parameter, the baryon-to-photon ratio η. Now, the evidence for neutrino mass from the Super-Kamiokande atmospheric neutrino observations is overwhelming [1]. In addition, the solar neutrino deficit and the LSND experiment [2] are hinting at the presence of a fourth light neutrino that would necessarily be "sterile" due to the Z decay width [3]. The role of neutrino masses and their mixing bring another aspect to the physical evolution of the Early Universe, when neutrinos played a large role. The effects of neutrinos on big bang nucleosynthesis can be in part parameterized by the effective number of neutrinos N_ν (*i.e.* the expansion rate), and in the weak physics which determine the neutron-to-proton ratio. In many works N_ν is regarded as the sole determinant of neutrino effects in BBN. However, we find that the effect of neutrino mixing on BBN is not well described by this one parameter. Characterizing the effects of neutrino mixing on BBN by N_ν can become overly complicated, since the parameter has been used to describe several distinct physical effects, including lepton number asymmetry and change in energy density. Another feature we find is that matter-enhanced mixing of neutrinos in the Early Universe can alter ν_e spectra to be non-thermal, which uniquely affects the nucleon weak rates by lifting Fermi blocking of neutron decay by neutrinos and suppression of neutrino capture on neutrons.

The observation of the abundances of primordial elements has greatly increased in precision within the past few years [4–8]. In SBBN, each observed primordial abundance corresponds to a value of the baryon-to-photon ratio $\eta \approx 2.79 \times 10^8\, \Omega_b^{-1} h^{-2}$, where Ω_b is the fractional contribution of baryon rest mass to the closure density and h is the Hubble parameter in units of $100\mathrm{km\,s^{-1}\,Mpc^{-1}}$. The theory is apparently successful since the inferred primordial abundances correspond within errors to a single value of η. SBBN took a new turn in 1996 when it was found that the deuterium abundance at high redshifts may be significantly lower than that inferred through the cosmic helium abundance [9]. The advent of high-resolution spectroscopy of quasars allowed the measurement of deuterium abundance in high-redshift clouds between us and distant quasars. The "early days" of these high-redshift spectral deuterium measurements were filled with controversy due to a disparity in the inferred deuterium to hydrogen (D/H) ratio between two observational groups working with different quasars. More recently, the number of quasars with low D/H has grown. The deuterium abundance has converged to ~10% precision: $(\mathrm{D/H})_p \simeq (3.4 \pm 0.25) \times 10^{-5}$. The η value inferred from this observation is $\eta_D \simeq (5.1 \pm 0.5) \times 10^{-10}$. The prediction for the corresponding primordial ^{4}He mass fraction is $Y_p(D) \simeq 0.246 \pm 0.0014$.

Based on stellar absorbtion features in metal-poor HII regions, the observed helium abundance has been argued by Olive, Steigman, and Skillman (OSS) [7] to be $Y_p(OSS) \simeq 0.234 \pm 0.002(\mathrm{stat}) \pm 0.005(\mathrm{sys})$. The corresponding baryon-to-photon ratio is $\eta_{OSS} \simeq (2.1 \pm 0.6) \times 10^{-10}$. The discrepancy between the central values of $Y_p(D)$ predicted by the Burles and Tytler deuterium observations and $Y_p(OSS)$ has led to much speculation on non-standard BBN scenarios which could reconcile the two measurements by decreasing the $Y_p(D)$ predicted by BBN. However, the systematic and statistical uncertainty in the $Y_p(D)$ and $Y_p(OSS)$ puts them well within their 2σ errors. Furthermore, Izotov and Thuan [10] exclude a few potentially tainted HII regions used in OSS from their analysis and arrive at a significantly higher $Y_p \simeq 0.244 \pm 0.002(\mathrm{stat})$. The systematic uncertainty claimed in observed Y_p may also be underestimated [11].

It has been suggested that this dubious Y_p and D/H discrepancy with standard BBN could be hinting at the presence of sterile neutrino mixing in the Early Universe [12]. However, we find that the overall effect of active-sterile mixing in the Early Universe would only increase Y_p and thus increase the possible disparity.

Active-sterile mixing can produce an electron lepton number asymmetry $L(\nu_e)$ before and during BBN [12,13]. If the sign of $L(\nu_e)$ is the same throughout the Universe, the helium abundance would change to be more favorable ($\delta Y_p < 0$) or less favorable ($\delta Y_p > 0$) with observation. Taking into account the effects of energy density, lepton number generation and alteration of neutrino spectra, we find that δY_p can be large in the positive direction, but limited in the negative direction [14]. In particular, we find that matter-enhanced transformations can leave non-thermal ν_e spectra which significantly affect the weak rates through the alleviation of Fermi-blocking and suppression of neutrino capture on neutrons [15]. Furthermore, causality limits the size of the region with a certain sign of $L(\nu_e)$ to be within the size of the horizon during these early times ($\sim 10^{10}$ cm at weak freeze-out) [16]. The sign of the asymmetry will vary between regions, and the net effect of transformation on Y_p will then be the average of both the $L(\nu_e) > 0$ and $L(\nu_e) < 0$ BBN yields, which leaves $\delta Y_p > 0$.

II. MATTER-ENHANCED MIXING OF NEUTRINOS AND NEUTRINO SPECTRA IN THE EARLY UNIVERSE

Two possible schemes exist for producing a nonzero $L(\nu_e)$ involving matter-enhanced (MSW) transformation of neutrinos with sterile neutrinos. These two schemes differ greatly in the energy distribution of the asymmetry in the spectra of the $\nu_e/\bar{\nu}_e$ the asymmetry. The resonant transformation of one neutrino flavor to another depends on the energy of the neutrino. In the case of BBN, the active neutrinos all start off as thermal, and sterile neutrinos are not present. As the Universe expands, decreases in density and cools, the position of the MSW resonance moves in energy space from lower to higher neutrino energies. The energy of the resonance is [14]

$$\left(\frac{E}{T}\right)_{\mathrm{res}} \approx \frac{|\delta m^2|/\mathrm{eV}^2}{16(T/\mathrm{MeV})^4 L(\nu_\alpha)} \tag{1}$$

where $(E/T)_{\mathrm{res}}$ is the neutrino energy normalized by the ambient temperature T. The asymmetry $L(\nu_\alpha)$ is generated as the resonance energy moves up the neutrino spectrum. As the Universe cools, the resonance energy increases. This produces a distortion of the neutrino or anti-neutrino spectrum. There are two plausible cases for producing an asymmetry $L(\nu_e)$ that would affect the production of primordial Helium through the weak rates:

- **Direct Two Neutrino Mixing:** An asymmetry in $L(\nu_e)$ is created directly through a $\nu_e \to \nu_s$ or $\bar{\nu}_e \to \bar{\nu}_s$ resonance. In this case, the resonance starts at low temperatures for low neutrino energies, when neutrino-scattering processes are very slow at re-thermalization. This case leaves the ν_e or $\bar{\nu}_e$ spectrum distorted, with the spectral distortion from a thermal Fermi-Dirac form residing in the low energy portion of the spectrum. The position of the distortion cut-off moves through the spectrum before and during BBN. In calculating the effects of this scenario on BBN, we included the evolving non-thermal nature of the neutrino spectrum.

- **Indirect Three Neutrino Mixing:** An asymmetry in $L(\nu_\tau)$ or $L(\nu_\mu)$ created by a $\nu_\tau \to \nu_s(\nu_\mu \to \nu_s)$ resonance is later transferred into the $L(\nu_e)$ through a $\nu_\tau \to \nu_e(\nu_\mu \to \nu_e)$ resonance ($L(\nu_e) > 0$). This will happen for the anti-neutrino flavors for the $L(\nu_e) < 0$ case. In this scenario, the resonant transitions occur at higher temperatures when neutrino-scattering is more efficient. Both the $\nu_\tau(\nu_\mu)$ and ν_e spectra re-thermalize, but since the temperature is below weak decoupling, the asymmetry in neutrino lepton numbers remain. The effect of this scenario on BBN can be described by suppression or enhancement over the entire $\nu_e/\bar{\nu}_e$ spectra.

III. NEUTRON-PROTON INTERCONVERSION RATES AND NEUTRINO SPECTRA

BBN can be approximated as the freeze-out of nuclides from nuclear statistical equilibrium in an "expanding box." Nuclear reactions freeze-out, or fail to convert one nuclide to another when their rate falls below the expansion rate of the box (the Hubble expansion). As different reactions rates freeze-out at different temperatures, the abundances of various nuclides get altered and then become fixed. The nuclide produced in greatest abundance (other than hydrogen) is ^{4}He. The reactions that affect the ^{4}He abundance (Y_p) the most are the weak nucleon interconversion reactions

$$n + \nu_e \leftrightarrow p + e^- \qquad n + e^+ \leftrightarrow p + \bar{\nu}_e \qquad n \leftrightarrow p + e^- + \bar{\nu}_e. \tag{2}$$

These reactions all depend heavily on the density of the electron-type neutrinos and their energy distributions [17]. The two neutrino transformation scenarios for altering the ν_e spectrum described above affect Y_p through these rates and through the effect of increased energy density on the expansion rate.

A. Direct Two-Neutrino Mixing: $\nu_s \leftrightarrow \nu_e$

In this situation, either the populations of the ν_e or $\bar{\nu}_e$ energy spectra are cutoff at lower energies. The reaction rates that alter the overall $n \leftrightarrow p$ rates the most because of the spectral cutoff are

$$\lambda_{n\to pe\nu} = A \int v_e E_\nu^2 E_e^2 dp_\nu [1 + e^{-E_\nu/kT_\nu}]^{-1} [1 + e^{-E_e/kT_\nu}]^{-1} \tag{3}$$

$$\lambda_{n\nu\to pe} = A \int v_e E_e^2 p_\nu^2 dp_\nu [e^{E_\nu/kT_\nu} + 1]^{-1} [1 + e^{-E_e/kT_\nu}]^{-1} \tag{4}$$

(the notation in these expressions follows that in Ref. [18]). The rate in Eq. 3 is modified with the $\bar{\nu}_e$ spectra, and the rate in Eq. 4 is modified with the ν_e spectra.

For a spectral cutoff for $\bar{\nu}_e$, corresponding to $\bar{\nu}_s \leftrightarrow \bar{\nu}_e$ transformations, the Fermi-blocking term $[1 + e^{-E_\nu/kT_\nu}]^{-1}$ in neutron decay (Eq. 3) becomes unity at low $\bar{\nu}_e$ energies. The rate integrand is then greatly enhanced when low energy neutrinos would otherwise block the process (see Figure 1). In the case of a spectral cutoff for ν_e, corresponding to $\nu_s \leftrightarrow \nu_e$ transformations, the spectral term $p_\nu^2[e^{E_\nu/kT_\nu} + 1]^{-1}$ and thus the rate integrand go to zero for low energy neutrinos (see Fig 2).

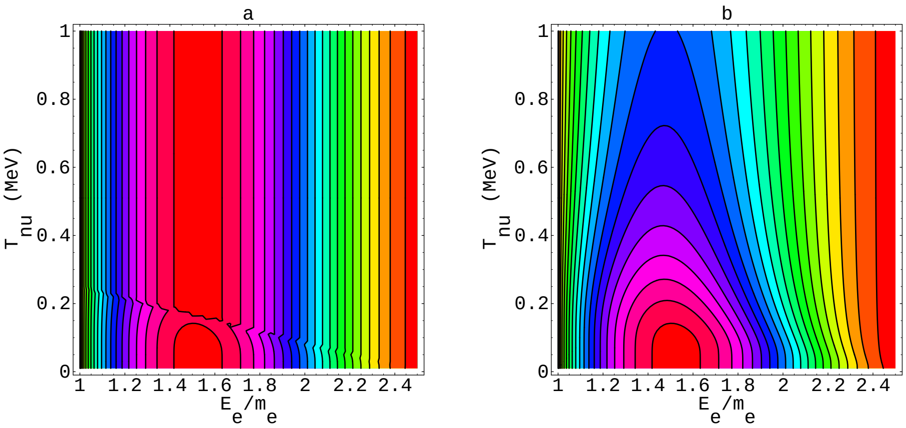

FIG. 1. Plotted are the contours of the amplitude of the rate integrand for $n \to p + e^- + \bar{\nu}_e$, with increasing amplitude towards the center. (a) shows the lack of fermi blocking by low energy electron anti-neutrinos at higher temperatures. (b) shows the standard BBN integrand.

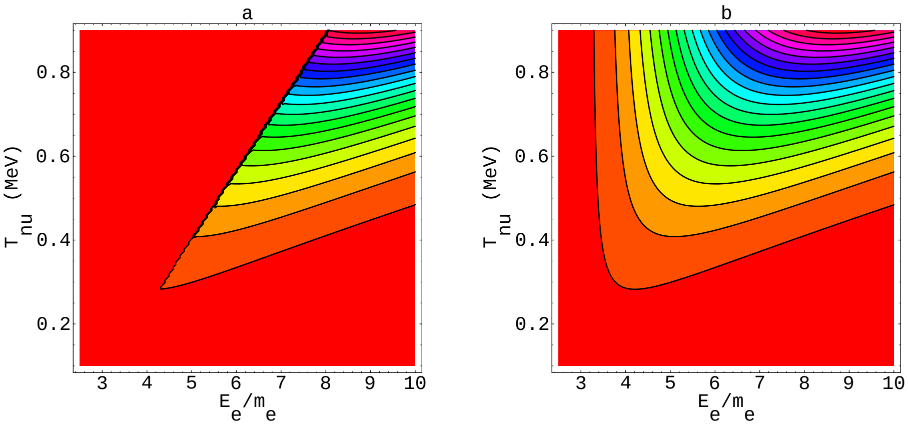

FIG. 2. Plotted are the contours of the amplitude of the rate integrand for $n + \nu_e \to p + e^-$, with increasing amplitude towards the upper right: (a) shows the blocking of the integrand for lower energies due to the suppression of low energy electron neutrinos; (b) shows the standard BBN integrand.

The resultant change in Y_p for this "two neutrino" case is shown in Figure 3. For $\bar{\nu}_e \to \bar{\nu}_s$ transformations, $L(\nu_e) > 0$, the neutron decay rate in Eq. 3 is enhanced, fewer neutrons are present, and Y_p is decreased. For $\nu_e \to \nu_s$ transformations, $L(\nu_e) < 0$, the $n\,\nu_e \to p\,e$ rate in Eq. 4 is suppressed, more neutrons are present, and Y_p is increased. The effect of the rate in Eq. 4 is much greater than that of the rate in Eq. 3 on the neutron-to-proton ratio and Y_p. This causes a larger magnitude change in the neutron-to-proton ratio for the $L(\nu_e) < 0$ case, and thus a larger magnitude change in Y_p in the positive direction than in the negative.

The sign of the asymmetry $L(\nu_e)$ is randomly determined [13]. Because of causality, different horizons will have an asymmetry of random sign [16], fifty-percent positive and fifty-percent negative. The effect of the asymmetry on Y_p will then be the average of these two cases, which is also plotted in Figure 3.

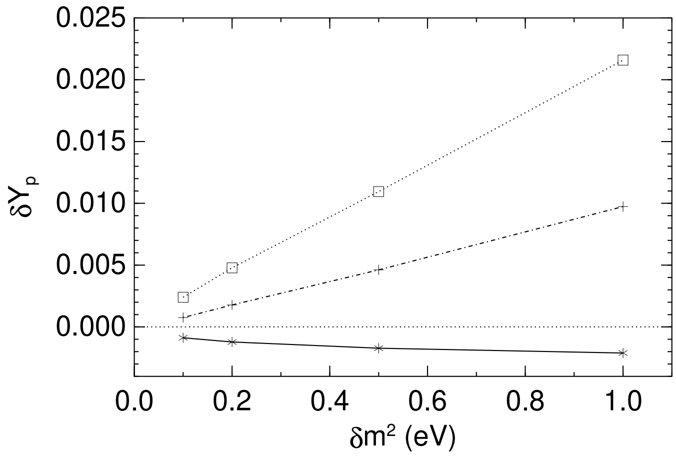

FIG. 3. Plotted are δY_p vs. δm^2 for the direct two-neutrino transformation scenario. The top line is for $L(\nu_e) < 0$; the bottom is for $L(\nu_e) > 0$; the middle line is the average of both cases enforced by causality.

B. Indirect Three-Neutrino Mixing

In the case that an asymmetry is produced in $\nu_e/\bar{\nu}_e$ indirectly through an initial transformation between $\nu_\tau \to \nu_s$ or $\nu_\mu \to \nu_s$, the neutrinos can re-thermalize through self-scattering at higher temperatures. The number densities remain asymmetric, but the asymmetry is spread uniformly over the entire energy spectra. The effect on the weak rates is not as drastic as the above case, but is still present. In this case, the increase in energy density due to the thermalization of sterile neutrinos also affects Y_p through the increase in the expansion rate. Thus, for $L(\nu_e) > 0$, the suppression of Y_p is counteracted by the increased expansion rate. The increase in energy density always causes a positive change in Y_p, while the change caused by the weak nucleon rates depends on the sign of $L(\nu_e)$, thus the overall resulting change in Y_p is greater in the positive direction than in the negative (see Figure 4). For the $L(\nu_e) > 0$ case, the negative change δY_p has a minimum at $\delta m^2 \approx 100\,\mathrm{eV}^2$ since at this point the positive effect of energy density begins to dominate the negative effect of the modification of the weak rates.

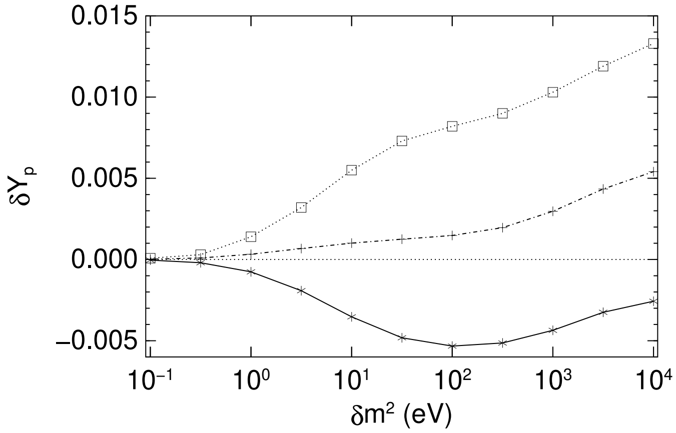

FIG. 4. Plotted are δY_p vs. δm^2 for the indirect three-neutrino transformation scenario. The top line is for $L(\nu_e) < 0$; the bottom is for $L(\nu_e) > 0$; the middle line is the average of both cases enforced by causality.

IV. CONCLUSIONS

The characterization of the effects of neutrino transformations on BBN through a single parameter N_ν hides varied physical effects of neutrino mass and mixing that are non-trivially related. These effects include an increase in energy density and modification in the nucleon weak rates. The modification of these rates is through either an asymmetry that is distributed over the entire spectrum, or an asymmetry confined to low energy $\nu_e/\bar{\nu}_e$. We find that the distribution of the $L(\nu_e)$ in the spectra is important to the weak rates, the neutron-to-proton ratio, and the production of ^{4}He. In addition, the change in the predicted Y_p is always greater in the positive direction than in the negative, thus when averaged over causal horizons, δY_p is always positive. The changes δY_p are comparable to the uncertainty in current Y_p measurements, thus resonant neutrino mixing can measurably affect the BBN predictions. It is important to note that there may remain non-trivial portions of parameter space in non-standard BBN—including, but not limited to, neutrino mixing, spatial variations in η, and massive decaying particles—that fit within the observed uncertainties of the primordial abundances.

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