PQCD analysis of exclusive B meson decays *

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The perturbative QCD formalism for exclusive heavy meson decays, especially the three-scale factorization theorem for nonleptonic modes is reviewed and compared to the conventional Bauer-Stech-Wirbel model.

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I. INTRODUCTION

Recently, perturbative QCD (PQCD) has been proposed to be an alternative approach to the study of heavy hadron decays [1–3], which complements the heavy quark effective theory (HQET) [4] for inclusive processes and the Bauer-Stech-Wirbel (BSW) method [5] for exclusive processes. The basic idea is the factorization theorem, which states that nonperturbative dynamics involved in a physical quantity can be factorized into a hadron distribution function or wave function. The remaining part, if characterized by a large scale, is calculable in perturbation theory. The distribution (wave) function, though not calculable, is universal. A physical quantity is then expressed as the convolution of perturbative parts with nonperturbative distribution (wave) functions. Once a distribution (wave) function is determined, say, from experimental data of some processes, it can be employed to make predictions of other processes involving the same hadron.

Within the PQCD framework, we have been able to explained the semileptonic branching ratio $B_{\rm SL}$ and the average charm yield n_c in inclusive B meson decays simultaneously [6]. Extending the same formalism to inclusive Λ_b baryon decays straightforwardly, we have predicted a low lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ [7], that the HQET approach can not achieve. For exclusive B meson decays, various transition form factors at large recoil, and factorizable and nonfactorrizable contributions can be evaluated systematically [8,9]. In this talk I will review the PQCD analysis of exclusive B meson decays, concentrating on the three-scale factorization theorem for nonleptonic modes [8], and explore the relation beetween the PQCD formalism and the BSW model.

II. THREE-SCALE FACTORIZATION THEOREMS

Nonleptonic B meson decays involve three scales: the W boson mass M_W (the matching scale), at which the matching conditions of the effective Hamiltonian to the full Hamiltonian are defined, the hard gluon momentum t of order the B meson mass M_B, which reflects the specific dynamics of different decay modes, and the factorization scale $1/b$ of order the QCD scale $\Lambda_{\rm QCD}$, b being the conjugate variable of parton transverse momenta. Dynamics below $1/b$ is regarded as being completely nonperturbative, and parametrized into a meson wave fucntion $\phi(x)$, x being the momentum fraction. Dynamics above the scale $1/b$ is perturbative, and absorbed into a hard subamplitude $H(t)$, if it is characterized by t, and into a "harder" function $H_r(M_W)$, if it is characterized by M_W. Semeleptonic decays depend only on the scales t and $1/b$, and thus their factorization involves only hard subamplitudes and meson wave functions.

Radiative corrections produce two types of large logarithms $\ln(M_W/t)$ and $\ln(tb)$. The former are summed to give the Wilson evolution $C(t)$ from M_W down to t, that connects the harder function and the hard subamplitude. While the latter are summed to give the renormalization-group (RG) evolution , (t,b) from t to $1/b$, that connects the hard subamplitude and the meson wave functions. There exist also double logarithms $\ln^2(Pb)$ from the overlap of collinear and soft divergences, P being the dominant light-cone component of a meson momentum. The resummation of these double logarithms leads to an exponential $\exp[-s(P,b)]$, which suppresses the long-distance contributions in the large

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b region, and improves the applicability of PQCD around the energy scale of few GeV [10]. The b quark mass scale is located in the range of applicability.

For the harder function, characterized by the large scale M_W, we adopt its lowest-order expression $H_r(M_W) = H_r^{(0)} = 1$. Because of Sudakov suppression, the hard subamplitude $H(t)$ can be evaluated reliably in perturbation theory. Note that $H(t)$ contains both factorizable and nonfactorizable contributions, each of which includes the types of external W-emissions, internal W-emissions, and W-exchanges. Take lowest-order diagrams as an example. If a hard gluon attaches the valence quarks of the same meson, the diagram gives factorizable contribution. If a hard gluon attaches the valence quarks of different mesons, the diagram gives nonfactorizable contribution. The Wilson coefficients $C(t)$ and the Sudakov factor $S(P,t,b) = \exp[-s(P,b)]$, (t,b) for heavy meson decays up to the accuracy of next-to-leading orders have been derived in [11] and in [1,2], respectively. Therefore, a three-scale factorization formula possesses the typical expression,

$$C(t) \otimes H(t) \otimes \phi(x) \otimes S(P,t,b) \;, \tag{1}$$

where all the convolution factors except the meson wave function ϕ are calculable.

Nonperturbative wave functions, though not calculable, are universal. They absorb the long-distance dynamics of a decay process, which are insensitive to the short-distance dynamics involved in the specific decay of the b quark into light quarks with large energy release. The universality of nonperturbative wave functions is the fundamental concept of PQCD factorization theorems. Because of the universality, the strategy of the PQCD approach is as follows: evaluate all perturbative factors for some decay modes, and adjust the wave functions such that predictions from the corresponding factorization formulas match experimental data. At this stage, the nonperturbative wave functions are determined up to the twists and orders of the coupling constant, at which the factorization formulas are constructed. Then evaluate the perturbative factors for another decay mode. Input the extracted wave functions into the factorization formulas of the same twist and orders, and make predictions. With this strategy, PQCD factorization theorems are model independent and possess a predictive power.

III. THE $B \to D^{(*)}\pi$ DECAYS

We take the nonleptonic decays $B \to D^{(*)}\pi$ as an example of the application of the three-scale factorization theorem. The decay rates of $B \to D^{(*)}\pi$ have the expression

$$, _i = \frac{1}{128\pi} G_F^2 |V_{cb}|^2 |V_{ud}|^2 M_B^3 \frac{(1-r^2)^3}{r} |\mathcal{M}_i|^2 \;, \tag{2}$$

where $i = 1$, 2, 3, and 4 denote the modes $B^- \to D^0\pi^-$, $\bar{B}^0 \to D^+\pi^-$, $B^- \to D^{*0}\pi^-$, and $\bar{B}^0 \to D^{*+}\pi^-$, respectively. The decay amplitudes $\mathcal{M}_i$ are written as

$$\mathcal{M}_1 = f_\pi[(1+r)\xi_+ - (1-r)\xi_-] + f_D\xi_{\rm int} + \mathcal{M}_{\rm ext} + \mathcal{M}_{\rm int} \;, \tag{3}$$
$$\mathcal{M}_2 = f_\pi[(1+r)\xi_+ - (1-r)\xi_-] + f_B\xi_{\rm exc} + \mathcal{M}_{\rm ext} + \mathcal{M}_{\rm exc} \;, \tag{4}$$
$$\mathcal{M}_3 = \frac{1+r}{2r} f_\pi[(1+r)\xi_{A_1} - (1-r)(r\xi_{A_2} + \xi_{A_3})] + f_{D^*}\xi^*_{\rm int} + \mathcal{M}^*_{\rm ext} + \mathcal{M}^*_{\rm int} \;, \tag{5}$$
$$\mathcal{M}_4 = \frac{1+r}{2r} f_\pi[(1+r)\xi_{A_1} - (1-r)(r\xi_{A_2} + \xi_{A_3})] + f_B\xi^*_{\rm exc} + \mathcal{M}^*_{\rm ext} + \mathcal{M}^*_{\rm exc} \;, \tag{6}$$

where f_B, $f_{D^{(*)}}$, and f_π are the B meson, $D^{(*)}$ meson, and pion decay constants, respectively. The form factors ξ_i, $i = +, -, V, A_1, A_2$, and A_3, denote the factorizable external W-emission contributions. The form factors $\xi^{(*)}_{\rm int}$ and $\xi^{(*)}_{\rm exc}$ denote the factorizable internal W-emission and W-exchange contributions, respectively. The amplitudes $\mathcal{M}^{(*)}_{\rm ext}$, $\mathcal{M}^{(*)}_{\rm int}$, and $\mathcal{M}^{(*)}_{\rm exc}$ represent the nonfactorizable external W-emission, internal W-emission, and W-exchange contributions, respectively.

The momenta of the B and $D^{(*)}$ mesons in light-cone coordinates are written as $P_1 = (M_B/\sqrt{2})(1,1,\mathbf{0}_T)$ and $P_2 = (M_B/\sqrt{2})(1,r^2,\mathbf{0}_T)$, respectively, with $r = M_{D^{(*)}}/M_B$. We define the momenta of light valence quarks in the B and $D^{(*)}$ mesons as k_1 and k_2, respectively. k_1 has a minus component k_1^-, giving the momentum fraction $x_1 = k_1^-/P_1^-$, and small transverse components $\mathbf{k}_{1T}$. k_2 has a large plus component k_2^+, giving $x_2 = k_2^+/P_2^+$, and small $\mathbf{k}_{2T}$. The pion momentum is then $P_3 = P_1 - P_2$, whose nonvanishing component is only P_3^-.

The resummation of the large logarithmic corrections to the meson wave functions leads to the exponentials,

$$S_B(t) = \exp\left[-s(x_1P_1^-, b_1) - 2\int_{1/b_1}^{t}\frac{d\bar\mu}{\bar\mu}\gamma(\alpha_s(\bar\mu))\right] , \quad (7)$$

$$S_{D^{(*)}}(t) = \exp\left[-s(x_2P_2^+, b_2) - s((1-x_2)P_2^+, b_2) - 2\int_{1/b_2}^{t}\frac{d\bar\mu}{\bar\mu}\gamma(\alpha_s(\bar\mu))\right] , \quad (8)$$

$$S_\pi(t) = \exp\left[-s(x_3P_3^-, b_3) - s((1-x_3)P_3^-, b_3) - 2\int_{1/b_3}^{t}\frac{d\bar\mu}{\bar\mu}\gamma(\alpha_s(\bar\mu))\right] . \quad (9)$$

The variable b_i, $i = 1$, 2, and 3, conjugate to the parton transverse momentum k_{iT}, represents the transverse extent of the corresponding mesons. The exponential with the exponent involving the anomalous dimension $\gamma = -\alpha_s/\pi$ corresponds to the RG evolution , mentioned above. For the explicit expression of the exponent s, refer to [12,13].

We present only the factorization formulas for the nonfactorizable amplitudes here. Those for factorizable contributions are referred to [14]. The integration over b_3 can be performed trivially, leading to $b_3 = b_1$ or $b_3 = b_2$. Their expressions are

$$\begin{aligned}
\mathcal{M}_{\rm ext}^{(*)} &= 32\pi\sqrt{2N}C_F\sqrt{r}M_B^2\int_0^1[dx]\int_0^\infty b_1db_1b_2db_2\phi_B(x_1)\phi_{D^{(*)}}(x_2)\phi_\pi(x_3)\frac{C_2(t_b)}{N}S(t_b)|_{b_2=b_1,b_3=b_2}\\
&\times\alpha_s(t_b)\left\{[x_3(1-r^2)-x_1-\zeta_b^{(*)}x_2(r-r^2)]h_b^{(1)}(x_i,b_i)-[x_3(1-r^2)-x_1+x_2]h_b^{(2)}(x_i,b_i)\right\} , \quad (10)\\
\mathcal{M}_{\rm int}^{(*)} &= 32\pi\sqrt{2N}C_F\sqrt{r}M_B^2\int_0^1[dx]\int_0^\infty b_1db_1b_2db_2\phi_B(x_1)\phi_{D^{(*)}}(x_2)\phi_\pi(x_3)\frac{C_1(t_d)}{N}S(t_d)|_{b_3=b_1}\\
&\times\alpha_s(t_d)\left\{[x_1-x_2-x_3(1-r^2)]h_d^{(1)}(x_i,b_i)-[(x_1+x_2)(1+\zeta_d^{(*)}r^2)-1]h_d^{(2)}(x_i,b_i)\right\} , \quad (11)\\
\mathcal{M}_{\rm exc}^{(*)} &= 32\pi\sqrt{2N}C_F\sqrt{r}M_B^2\int_0^1[dx]\int_0^\infty b_1db_1b_2db_2\phi_B(x_1)\phi_{D^{(*)}}(x_2)\phi_\pi(x_3)\frac{C_1(t_f)}{N}S(t_f)|_{b_3=b_2}\\
&\times\alpha_s(t_f)\left\{[x_3(1-r^2)-\zeta_f^{(*)}(x_1-x_2)r^2]h_f^{(1)}(x_i,b_i)-[(x_1+x_2)(1+\zeta_f^{(*)}r^2)-\zeta_f^{(*)}r^2]h_f^{(2)}(x_i,b_i)\right\} , \quad (12)
\end{aligned}$$

with the number of color $N = 3$, the color factor $C_F = 4/3$, the definition $[dx] \equiv dx_1dx_2dx_3$, and the constants $\zeta_{b,d,f} = -\zeta^*_{b,d,f} = 1$. The Wilson coefficients $C_{1,2}$ will be defined later. The complete Sudakov factor is given by the product of Eqs. (7)-(9), $S = S_BS_{D^{(*)}}S_\pi$.

The functions $h^{(j)}$, $j = 1$ and 2, appearing in Eqs. (10)-(12), are written as

$$\begin{aligned}
h_b^{(j)} &= [\theta(b_1-b_2)K_0(BM_Bb_1)I_0(BM_Bb_2)+\theta(b_2-b_1)K_0(BM_Bb_2)I_0(BM_Bb_1)]\\
&\times\begin{pmatrix} K_0(B_jM_Bb_2) & \text{for } B_j^2\geq 0\\ \frac{i\pi}{2}H_0^{(1)}(\sqrt{|B_j^2|}M_Bb_2) & \text{for } B_j^2\leq 0\end{pmatrix} , \quad (13)\\
h_d^{(j)} &= [\theta(b_1-b_2)K_0(DM_Bb_1)I_0(DM_Bb_2)+\theta(b_2-b_1)K_0(DM_Bb_2)I_0(DM_Bb_1)]\\
&\times\begin{pmatrix} K_0(D_jM_Bb_2) & \text{for } D_j^2\geq 0\\ \frac{i\pi}{2}H_0^{(1)}(\sqrt{|D_j^2|}M_Bb_2) & \text{for } D_j^2\leq 0\end{pmatrix} , \quad (14)\\
h_f^{(j)} &= i\frac{\pi}{2}\left[\theta(b_1-b_2)H_0^{(1)}(FM_Bb_1)J_0(FM_Bb_2)+\theta(b_2-b_1)H_0^{(1)}(FM_Bb_2)J_0(FM_Bb_1)\right]\\
&\times\begin{pmatrix} K_0(F_jM_Bb_1) & \text{for } F_j^2\geq 0\\ \frac{i\pi}{2}H_0^{(1)}(\sqrt{|F_j^2|}M_Bb_1) & \text{for } F_j^2\leq 0\end{pmatrix} , \quad (15)
\end{aligned}$$

with the variables

$$\begin{aligned}
B^2 &= x_1x_2 ,\\
B_1^2 &= x_1x_2 - x_2x_3(1-r^2) ,\\
B_2^2 &= x_1x_2(1+r^2) - (x_1-x_2)(1-x_3)(1-r^2) ,\\
D^2 &= x_1x_3(1-r^2) ,\\
D_1^2 &= F_1^2 = (x_1-x_2)x_3(1-r^2) ,
\end{aligned}$$

$$D_2^2 = (x_1+x_2)r^2 - (1-x_1-x_2)x_3(1-r^2) ,$$
$$F^2 = x_2 x_3(1-r^2) ,$$
$$F_2^2 = x_1 + x_2 + (1-x_1-x_2)x_3(1-r^2) . \tag{16}$$

The scales $t^{(j)}$ are chosen as

$$t_b = \max(BM_B, \sqrt{|B_1^2|}M_B, \sqrt{|B_2^2|}M_B, 1/b_1, 1/b_2) ,$$
$$t_d = \max(DM_B, \sqrt{|D_1^2|}M_B, \sqrt{|D_2^2|}M_B, 1/b_1, 1/b_2) ,$$
$$t_f = \max(FM_B, \sqrt{|F_1^2|}M_B, \sqrt{|F_2^2|}M_B, 1/b_1, 1/b_2) . \tag{17}$$

The wave functions $\phi_i(x)$, $i = B$, $D^{(*)}$, and π, satisfy the normalization

$$\int_0^1 \phi_i(x)dx = \frac{f_i}{2\sqrt{6}} , \tag{18}$$

with the corresponding decay constants f_i. It is easy to observe that Eqs. (10)-(12) agree with the typical expression in Eq. (1).

IV. COMPARISION TO THE BSW METHOD

In this section I compare the PQCD approach with the BSW model. Nonleptonic heavy meson decays occur through the Hamiltonian,

$$H = \frac{G_F}{\sqrt{2}} V_{ij} V_{kl}^* (\bar{q}_l q_k)(\bar{q}_j q_i) , \tag{19}$$

where G_F is the Fermi coupling constant, V_{cb} the Cabibbo-Kabayashi-Maskawa (CKM) matrix element, and $(\bar{q}q) = \bar{q}\gamma_\mu(1-\gamma_5)q$ the $V-A$ current. Hard gluon corrections cause an operator mixing, and their RG summation leads to the effective Hamiltonian,

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ij} V_{kl}^* [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] , \tag{20}$$

with the four-fermion operators $O_1 = (\bar{q}_l q_k)(\bar{q}_j q_i)$ and $O_2 = (\bar{q}_j q_k)(\bar{q}_l q_i)$. The matching conditions of the Wilson coefficients are given by $C_1(M_W) = 1$ and $C_2(M_W) = 0$.

The most widely adopted approach to exclusive nonleptonic heavy meson decays is the BSW model [5], in which the factorization hypothesis on the matrix elements of the operators $O_{1,2}$ is assumed. In this model decay rates are expressed in terms of various hadronic transition form factors. Employing Fierz transformation, the coefficient of the form factors corresponding to external W boson emissions is $a_1 = C_1 + C_2/N$, and that corresponding to internal W boson emissions is $a_2 = C_2 + C_1/N$. The form factors may be related to each other by heavy quark symmetry, and parametrized by different ansatz. Nonfactorizable contributions, which can not be expressed in terms of hadronic transition form factors, and nonspectator contributions from W boson exchanges are neglected.

Physical quantities such as decay amplitudes should not depend on the renormalization scale μ. In principle, the matrix elements of the four-fermion operators contain a μ dependence, which exactly cancels that of the Wilson coefficients. In the BSW method, however, nonleptonic matrix elements are factorized into two matrix elements of (axial) vector currents. Since the currents are conserved, the matrix elements have no anomalous scale dependence. Therefore, predictions from the BSW model are μ-dependent, and can not be physical. In the PQCD approach, the RG evolutions from M_W to t and from t to $1/b$ are taken into account, such that the factorization formulas are scale-independent.

To circumvent the scale dependence in the BSW model, the Wilson coefficients a_i are regarded as free parameters, and are determined by experimental data [5]. However, the evaluation of the hadronic form factors usually involve some ansatz [15], so that the extraction of a_1 and a_2 is model dependent. On the other hand, it was found that the ratio a_2/a_1 from an individual fit to the CLEO data of $B \to D^{(*)}\pi(\rho)$ [16] varies significantly [15]. It was also shown that an allowed domain (a_1, a_2) exists for the three classes of decays $\bar{B}^0 \to D^{(*)+}$, $\bar{B}^0 \to D^{(*)0}$ and $B^- \to D^{(*)0}$, only when the experimental errors are expanded to a large extent [17]. In the PQCD approach the Wilson coefficients, with their evolutions being determined by RG equations as shown in Eqs. (10)-(12), are not free universal parameters. The hard scale t depends on meson dynamics, and is thus process-dependent. The specific dynamics involved in different B meson decays are then reflected by the scale t, or equivalently, by the RG evolutions.

The BSW model encounters other difficulties. It has been known that the large N limit of $a_{1,2}$, *i.e.*, the choice $a_1 = C_1(M_c) \approx 1.26$ and $a_2 = C_2(M_c) \approx -0.52$, with M_c the c quark mass, explains the data of charm decays [5]. However, the same large N limit of $a_1 = C_1(M_b) \approx 1.12$ and $a_2 = C_2(M_b) \approx -0.26$, M_b being the b quark mass, does not apply to the bottom case. That is, the different mechanism between charm and bottom decays can not be understood in the BSW approach. To overcome this difficulty, parameters χ, denoting the corrections from the nonfactorizable contributions, have been introduced [18]. They lead to the effective coefficients

$$a_1^{\rm eff} = C_1 + C_2\left(\frac{1}{N} + \chi_1\right) , \quad a_2^{\rm eff} = C_2 + C_1\left(\frac{1}{N} + \chi_2\right) . \tag{21}$$

χ should be negative for charm decays, canceling the color-suppressed term $1/N$, and be positive for bottom decays in order to enhance the predictions. In the PQCD formalism the nonfactorizable contributions do not lead to additional parameters. They can be evaluated by convoluting the nonfactorizable hard subamplitude (calculable in perturbation theory) with the same meson wave functions as those for factorizable contributions because of the universality. Using the three-scale factorization formulas, the mechanism responsible for the sign change of χ has been found [19].

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