Theoretical Developments in Inclusive B Decays

Zoltan Ligeti
Theory Group, Fermilab, P.O. Box 500, Batavia, IL 60510

Some recent theoretical work on inclusive B decays relevant for the model independent determination of $|V_{ub}|$ and $|V_{cb}|$ is summarized. The theoretical predictions and their reliability for several differential decay distributions in $\bar{B} \to X_{c,u} e\bar{\nu}$ and $\bar{B} \to X_s\gamma$ are reviewed. These can be used to determine certain important HQET matrix elements. The upsilon expansion and ways of testing it are discussed.

I. INTRODUCTION

In the near future, a large part of the high energy experimental program will be devoted to testing the Cabibbo–Kobayashi–Maskawa (CKM) picture of quark mixing and CP violation by directly measuring the sides and (some) angles of the unitarity triangle. If the value of $\sin(2\beta)$, the CP asymmetry in $B \to J/\psi K_S$, is not too far from the CDF central value [1], then searching for new physics at the B factories will require a combination of several precision measurements. Particularly important are $|V_{ub}|$ and $|V_{td}|$, which are the least precisely known elements of the CKM matrix. The latter will be measured hopefully in the upcoming run of the Tevatron from the ratio of B_s and B_d mixing, and will not be discussed here. This talk is motivated by trying to understand what the chances are to

- ... reduce (conservative) error in $|V_{cb}|$ below $\sim 5\%$? Although sometimes a smaller error is quoted already (e.g., by the Particle Data Group), it is hard to bound model independently the possible quark-hadron duality violation in the inclusive, and the size of $1/m_{c,b}^2$ corrections in the exclusive determination.
- ... determine $|V_{ub}|$ with less than $\sim 10\%$ error? The inclusive measurements require significant cuts on the available phase space; the exclusive measurements require knowledge of the form factors.
- ... reduce the theoretical uncertainties in the $B \to X_s\gamma$ photon spectrum? The effect of the experimental cut on the photon energy needs to be better understood, and there are subtleties in the OPE beyond leading order.

The theoretical reliablility of inclusive measurements can be competitive with the exclusive ones (or even better in some cases). For example, for the determination of $|V_{cb}|$ model dependence enters at the same order of $\Lambda_{\rm QCD}^2/m^2$ corrections from both the inclusive semileptonic $\bar{B} \to X_c e\bar{\nu}$ width and the $\bar{B} \to D^* e\bar{\nu}$ rate near zero recoil.

Inclusive B decay rates can be computed model independently in a series in $\Lambda_{\rm QCD}/m_b$, using an operator product expansion (OPE) [2–4]. The $m_b \to \infty$ limit is given by b quark decay. For most quantities of interest, this result is known including the order α_s and the dominant part of the order α_s^2 corrections. Observables which do not depend on the four-momentum of the hadronic final state (e.g., total decay rate and lepton spectra) receive no correction at order $\Lambda_{\rm QCD}/m_b$ when written in terms of m_b, whereas differential rates with respect to hadronic variables (e.g., hadronic energy and invariant mass spectra) also depend on $\bar{\Lambda}/m_b$, where $\bar{\Lambda}$ is the $m_B - m_b$ mass difference in the $m_b \to \infty$ limit. At order $\Lambda_{\rm QCD}^2/m_b^2$, the corrections are parameterized by two hadronic matrix elements, usually denoted by λ_1 and λ_2. The value $\lambda_2 \simeq 0.12\,\text{GeV}^2$ is known from the $B^* - B$ mass splitting.

For inclusive $b \to q$ decay, corrections to the $m_b \to \infty$ limit are expected to be under control in parts of phase space where several hadronic final states are allowed to contribute with invariant masses satisfying $m_{X_q}^2 \gtrsim m_q^2 + (\text{few times})\Lambda_{\rm QCD} m_b$.[1] Such observables which "average" sufficiently over different hadronic final states may be predicted reliably. (We are just beginning to learn quantitatively what sufficient averaging is.) The major uncertainty in these predictions is from the values of the quark masses and λ_1, or equivalently, the values of $\bar{\Lambda}$ and λ_1. These quantities can be extracted from heavy meson decay spectra, which is the subject of a large part of this talk.

[1] However, it is not true that several such states are required to contribute; e.g., $\bar{B} \to X_c e\bar{\nu}$ decay in the small velocity limit can be computed reliably, even though it is saturated by D and D^* only [5,6].

An important theoretical subtlety is related to the fact that $\bar{\Lambda}$ cannot be defined unambiguously beyond perturbation theory [7], and its value extracted from data using theoretical expressions valid to different orders in the α_s may vary by order $\Lambda_{\rm QCD}$. However, these ambiguities cancel [8] when one relates consistently physical observables to one another, i.e., the formulae used to determine $\bar{\Lambda}$ and λ_1 contain the same orders in the α_s perturbation series as those in which the so extracted values are applied to make predictions.

II. THE HQET PARAMETERS $\bar{\Lambda}$ AND λ_1

The shape of the lepton energy [9–11] or hadronic invariant mass [12,13,11] spectrum in semileptonic $\bar{B} \to X_c\,\ell\,\bar{\nu}$ decay, and the photon spectrum in $\bar{B} \to X_s\gamma$ [14–18] can be used to measure the heavy quark effective theory (HQET) parameters $\bar{\Lambda}$ and λ_1. Testing our understanding of the $\bar{B} \to X_c e\bar{\nu}$ spectra is important to assess the reliability of the inclusive determination of $|V_{cb}|$, and especially that of $|V_{ub}|$. Understanding the photon spectrum in $\bar{B} \to X_s\gamma$ is important to evaluate how precisely the total rate can be predicted in the presence of an experimental cut on the photon energy [19]. Studying these distributions is also useful to establish the limitations of models which were built to fit the lepton energy spectrum in semileptonic decay.

The theoretical predictions are known to order $\alpha_s^2\beta_0$, where $\beta_0 = 11 - 2n_f/3$ is the coefficient of the one-loop β-function. This part of the order α_s^2 piece usually provides a reliable estimate of the full order α_s^2 correction, and it is straightforward to compute using the method of Smith and Voloshin [20]. In some cases the full α_s^2 correction is known. The order $\Lambda_{\rm QCD}^3/m_b^3$ terms have also been studied [11,13,17], and are used to estimate the uncertainties.

The OPE for semileptonic (or radiative) B decay does not reproduce the physical spectrum point-by-point in regions of phase space where the hadronic invariant mass of the final state is restricted. For example, the maximum electron energy for a particular hadronic final state X is $E_e^{(\rm max)} = (m_B^2 - m_X^2)/2m_B$, so comparison with experimental data near the endpoint can only be made after sufficient smearing, or after integrating over a large enough region. The minimal size of this region was estimated to be around $300 - 500\,{\rm MeV}$ [4]. The hadron mass spectrum cannot be predicted point-by-point without additional assumptions, but moments of it are calculable. In general, higher moments of a distribution or moments over smaller regions are less reliable than lower moments or moments over larger regions. The strategy is to find observables which are sensitive to $\bar{\Lambda}$ and λ_1, but the deviations from b quark decay are small, so that the contributions from operators with dimension greater than 5 are not too important.

A. $\bar{B} \to X_c e\bar{\nu}$ decay spectra

Last year the CLEO Collaboration measured the first two moments of the hadronic invariant mass-squared (s_H) distribution, $\langle s_H - \overline{m}_D^2\rangle$ and $\langle (s_H - \overline{m}_D^2)^2\rangle$, subject to the constraint $E_e > 1.5\,{\rm GeV}$ [21]. Here $\overline{m}_D = (m_D + 3m_{D^*})/4$. Each of these measurements give an allowed band on the $\bar{\Lambda} - \lambda_1$ plane. These bands are almost perpendicular, so they give the fairly small intersection region shown in Fig. 1. The central values at order α_s are [21]

$$\bar{\Lambda} = (0.33 \pm 0.08)\,{\rm GeV}\,, \qquad \lambda_1 = -(0.13 \pm 0.06)\,{\rm GeV}^2\,. \tag{1}$$

The unknown order $\Lambda_{\rm QCD}^3/m_b^3$ corrections not included in this result introduce a large uncertainty, especially for the second moment. As a result, the allowed range is much longer in the direction of the first moment band than perpendicular to it; see Fig. 2.

Similar information on $\bar{\Lambda}$ and λ_1 can be obtained from the lepton energy spectrum, $d\Gamma/dE_e$. It has been measured both by demanding only one charged lepton tag, and using a double tagged data sample where the charge of a high momentum lepton determines whether the other lepton in the event comes directly from semileptonic B decay (primary) or from the semileptonic decay of a B decay product charmed hadron (secondary). The single tagged data has smaller statistical errors, but it is significantly contaminated by secondary leptons below about 1.5 GeV. Therefore, Ref. [9] considered the observables

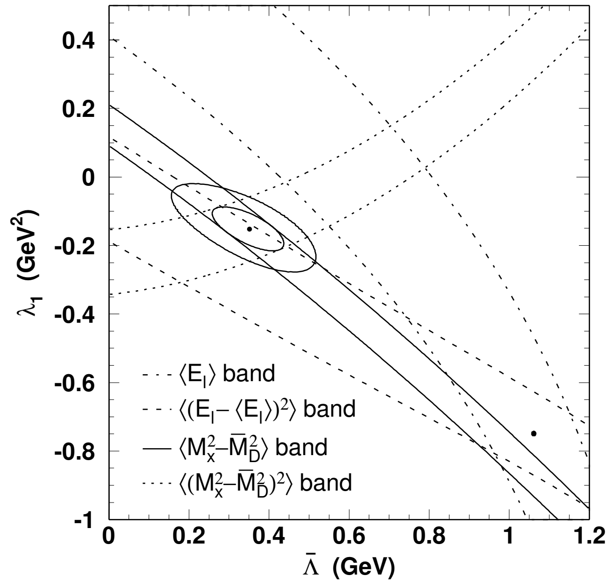

FIG. 1. Bands in the $\bar\Lambda - \lambda_1$ plane defined by the measured first and second moments of the hadronic mass-squared and lepton energy distributions. Note that $M_X^2 \equiv s_H$. (From Ref. [21].)

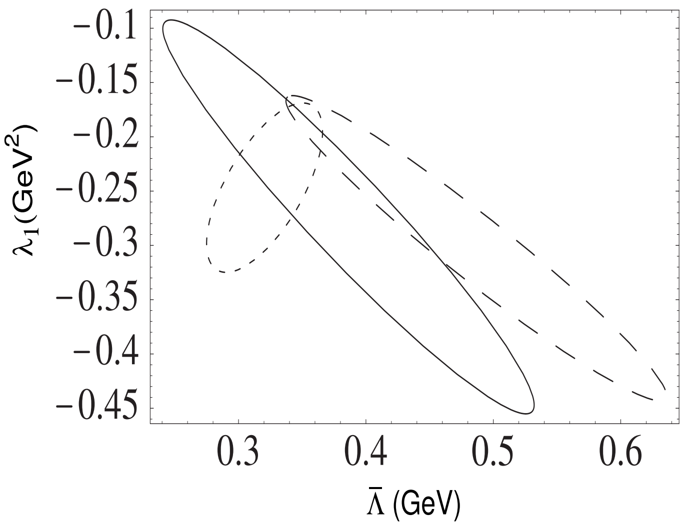

FIG. 2. Estimates of the theoretical uncertainty in $\bar\Lambda$ and λ_1 due to unknown $1/m_b^3$ terms from the shape of the electron spectrum (solid ellipse), from hadronic mass moments (long dashed ellipse), and the $\bar B \to X_s\gamma$ spectrum (short dashed ellipse; see Sec. IIB). Only the relative size and orientation of the ellipses are meaningful, not their position. (From Refs. [13] and [17].)

$$R_1 = \frac{\displaystyle\int_{1.5\,\mathrm{GeV}} E_\ell \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\displaystyle\int_{1.5\,\mathrm{GeV}} \frac{d\Gamma}{dE_\ell}\, dE_\ell}, \qquad R_2 = \frac{\displaystyle\int_{1.7\,\mathrm{GeV}} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\displaystyle\int_{1.5\,\mathrm{GeV}} \frac{d\Gamma}{dE_\ell}\, dE_\ell}\,. \tag{2}$$

Using the CLEO data [22], the central values $\bar\Lambda = 0.39\,\mathrm{GeV}$ and $\lambda_1 = -0.19\,\mathrm{GeV}^2$ were obtained [9], which is in good agreement with Eq. (1).

However, following Ref. [10], CLEO determined the first two moments of the spectrum, $\langle E_e \rangle$ and $\langle (E_e - \langle E_e \rangle)^2 \rangle$, without any restriction on E_e, using the double tagged data and an extrapolation to $E_e < 0.6\,\mathrm{GeV}$. The result of this analysis is also plotted in Fig. 1, and yields quite improbable values for $\bar\Lambda$ and λ_1. The extrapolation to $E_e < 0.6\,\mathrm{GeV}$ introduces unnecessary model dependence, so this result may be less reliable than the hadronic invariant mass analysis. The extracted values of $\bar\Lambda$ and λ_1 are very sensitive to small systematic effects; and it seems problematic for the exclusive models used for the extrapolation to simultaneously reproduce the inclusive lepton spectrum and the B semileptonic branching fraction [23].

There are a number of points to emphasize regarding these results:

1. Taking $\bar\Lambda$ from Eq. (1) gives a determination of $|V_{cb}|$ from semileptonic B width with $\sim 3\%$ uncertainty.
2. The inclusive $|V_{cb}|$ seems to be slightly larger than the exclusive (maybe not "significantly" but "consistently"). Can the theoretical uncertainties be reduced by combining the exclusive and inclusive determinations of $|V_{cb}|$?
3. Since the bands from the lepton energy spectrum and the first moment of the hadronic mass-squared spectrum are almost parallel, an independent constraint on the $\bar\Lambda - \lambda_1$ plane is needed. (See Section II B.)
4. Models do not seem to do well for $\bar B \to D^{**} e\bar\nu$ and $\bar B \to D^{(*)}\pi e\bar\nu$ — relying on them may be dangerous. Composition of semileptonic B decay seems to be not really understood yet. (See next subsection.)

1. Semileptonic B decays to excited charmed mesons

In the heavy quark symmetry limit [24], the spin and parity of the light degrees of freedom in a heavy meson are conserved. In the charm sector, the ground state is the $(D,\, D^*)$ doublet of heavy quark spin symmetry with spin-parity of the light degrees of freedom $s_\ell^{\pi_\ell} = \frac12^-$. The four lightest excited states, sometimes referred to as D^{**}, are the $(D_0^*,\, D_1^*)$ doublet with $s_\ell^{\pi_\ell} = \frac12^+$ and the $(D_1,\, D_2^*)$ doublet with $s_\ell^{\pi_\ell} = \frac32^-$. The D_1 and D_2^* have been observed with masses near 2.42 and 2.46 GeV, respectively, and width around 20 MeV. States in the $s_l^{\pi_l} = \frac12^+$ doublet can decay into $D^{(*)}\pi$ in an s-wave, and so they are expected to be much broader than the D_1 and D_2^* which can only decay in a d-wave. (An s-wave decay for the D_1 is forbidden by heavy quark spin symmetry [25].) The first observation of the D_1^* state with mass and width about 2.46 GeV and 290 MeV, respectively, was reported at this Conference [26].

$\bar B \to D_1 e\bar\nu$ and $\bar B \to D_2^* e\bar\nu$ account for sizable fractions of semileptonic B decays, and are probably the only three-body semileptonic B decays (other than $\bar B \to D^{(*)} e\bar\nu$) whose differential decay distributions will be precisely measured. ALEPH and CLEO measured recently, with some assumptions, $\mathcal{B}(\bar B \to D_1 e\bar\nu_e) = (6.0 \pm 1.1) \times 10^{-3}$ [27], while $\bar B \to D_2^* e\bar\nu_e$ has not been seen yet. Heavy quark symmetry implies that in the $m_Q \to \infty$ limit $(Q = c, b)$, matrix elements of the weak currents between a B meson and an excited charmed meson vanish at zero recoil. However, in some cases at order $\Lambda_{\rm QCD}/m_Q$ these matrix elements are not zero. Since most of the phase space for semileptonic B decay to excited charmed mesons is near zero recoil, $1 < v \cdot v' \lesssim 1.3$, $\Lambda_{\rm QCD}/m_Q$ corrections can be very important.

The matrix elements of the weak currents between B mesons and D_1 or D_2^* mesons are conventionally parameterized in terms of a set of eight form factors f_i and k_i [28]. At zero recoil only f_{V_1}, defined by

$$\langle D_1(v',\epsilon)|\, \bar c\, \gamma^\mu\, b\, |B(v)\rangle = \sqrt{m_{D_1}\, m_B}\, [f_{V_1}\, \epsilon^{*\mu} + (f_{V_2} v^\mu + f_{V_3} v'^\mu)\, (\epsilon^* \cdot v)]\,, \tag{3}$$

can contribute to the matrix elements. In the $m_Q \to \infty$ limit, f_i and k_i are given in terms of a single Isgur-Wise function, $\tau(w)$ [29]. Heavy quark symmetry does not fix $\tau(1)$, since $f_{V_1} = (1-w^2)\,\tau(w) + \mathcal{O}(1/m_Q)$, and so $f_{V_1}(1) = 0$ in the infinite mass limit independent of the value of $\tau(1)$.

At order $1/m_Q$ several new Isgur-Wise functions occur, together with the parameter $\bar\Lambda'$, which is the analog of $\bar\Lambda$ for the $(D_1,\, D_2^*)$ doublet. $\bar\Lambda' - \bar\Lambda \simeq 0.39$ GeV follows from the measured meson masses [28]. At order $1/m_Q$, $f_{V_1}(1)$ is no longer zero, but it can be written in terms of $\bar\Lambda' - \bar\Lambda$ and the Isgur-Wise function $\tau(w)$ evaluated at zero recoil [28],

$$\sqrt6\, f_{V_1}(1) = -\frac{4(\bar\Lambda' - \bar\Lambda)}{m_c}\, \tau(1)\,. \tag{4}$$

This relation means that at zero recoil heavy quark symmetry gives some model independent information about the $1/m_Q$ corrections, similar to Luke's theorem [30] for the decay into the ground state $(D,\, D^*)$ doublet.

Since the allowed kinematic range for $\bar B \to D_1 e\bar\nu_e$ and $\bar B \to D_2^* e\bar\nu_e$ decay are fairly small $(1 < w \lesssim 1.3)$, and there are some constraints on the $1/m_Q$ corrections at zero recoil, it is useful to consider the decay rates expanded in powers of $w - 1$. The general structure of the expansion of $d\Gamma/dw$ is elucidated schematically below,

$$\begin{aligned}
\frac{d\Gamma_{D_1}^{(\lambda=0)}}{dw} &\sim \sqrt{w^2-1}\, \Big[(0 + 0\,\varepsilon + \varepsilon^2 + \ldots) + (w-1)\,(0 + \varepsilon + \ldots) + (w-1)^2\,(1 + \varepsilon + \ldots) + \ldots\Big]\,, \\
\frac{d\Gamma_{D_1}^{(|\lambda|=1)}}{dw} &\sim \sqrt{w^2-1}\, \Big[(0 + 0\,\varepsilon + \varepsilon^2 + \ldots) + (w-1)\,(1 + \varepsilon + \ldots) + (w-1)^2\,(1 + \varepsilon + \ldots) + \ldots\Big]\,, \\
\frac{d\Gamma_{D_2^*}^{(|\lambda|=0,1)}}{dw} &\sim (w^2-1)^{3/2}\, \Big[(1 + \varepsilon + \ldots) + (w-1)(1 + \varepsilon + \ldots) + \ldots\Big]\,.
\end{aligned} \tag{5}$$

Here λ is the helicity of the D_1 or D_2^*, and ε^n denotes a term of order $(\Lambda_{\rm QCD}/m_Q)^n$. The zeros in Eq. (5) are consequences of heavy quark symmetry.

Table I summarizes some of the more important predictions. The first row (B_∞) shows the infinite mass limit. Approximations B_1 and B_2 are two different ways of treating unknown $1/m_Q$ corrections, and may give an indication of the uncertainty at this order. One of the most interesting predictions is that the $B \to D_1$ decay rate should be larger than $B \to D_2^*$, contrary to the infinite mass limit, but in agreement with the data. A number of other predictions for different helicity amplitudes, factorization of $\bar B \to (D_1,\, D_2^*)\pi e\bar\nu$ decay, sum rules, etc., are presented in Ref. [28] (see also [31]). The main points are:

1. At zero recoil, order $1/m_Q$ contributions to semileptonic $B \to D_1,\, D_2^*$ decays (any excited charmed meson with $+$ parity) are determined by the $m_Q \to \infty$ Isgur-Wise function and known hadron mass splittings.

Approximation	$R = \Gamma_{D_2^*}/\Gamma_{D_1}$	$\tau(1)\left[\dfrac{6.0\times 10^{-3}}{\mathcal{B}(\bar B\to D_1\, e\,\bar\nu_e)}\right]^{1/2}$	$\Gamma_{D_1+D_2^*+D_1^*+D_0^*}/\Gamma_{D_1}$
B_∞	1.65	1.24	4.26
B_1	0.52	0.71	2.55
B_2	0.67	0.75	2.71

TABLE I. Predictions for $\Gamma_{D_2^*}/\Gamma_{D_1}$, $\tau(1)$, and $\Gamma_{D_1+D_2^*+D_1^*+D_0^*}/\Gamma_{D_1}$. (From Ref. [28].)

2. Decay spectra can be predicted near zero recoil, including the $1/m_Q$ corrections, with reasonable assumptions.
3. Test heavy quark symmetry for B decays to excited charmed mesons, where the $1/m_Q$ terms are sometimes the leading contributions. Large $1/m_Q$ corrections to some predictions can be checked against data.
4. Better understanding of inclusive $=\sum$exclusive in semileptonic B decay. If semileptonic B decays into the four D^{**} states indeed account for less than 2% of the B width, then about another 2% of B decays must be semileptonic decays to higher mass excitations or nonresonant channels.
5. Constrain the validity of models constructed to fit the decays to the ground state final state (most predict, for example, larger $B\to D_2^*$ than $B\to D_1$ semileptonic rate).

B. $\bar B\to X_s\gamma$ photon spectrum

Comparison of the measured weak radiative $\bar B\to X_s\gamma$ decay rate with theory is an important test of the standard model. In contrast to the decay rate itself, the shape of the photon spectrum is not expected to be sensitive to new physics, but it can nevertheless provide important information. First of all, studying the photon spectrum is important for understanding how precisely the total rate can be predicted in the presence of an experimental cut on the photon energy [19], which is needed for a model independent interpretation of the resulting decay rate. Secondly, moments of the photon spectrum may be used to measure the HQET parameters $\bar\Lambda$ and λ_1 [14–18].

An important observable is

$$\overline{(1-x_B)}\Big|_{x_B>1-\delta} = \frac{\displaystyle\int_{1-\delta}^{1} dx_B\,(1-x_B)\,\frac{d\Gamma}{dx_B}}{\displaystyle\int_{1-\delta}^{1} dx_B\,\frac{d\Gamma}{dx_B}}\,, \tag{6}$$

where $x_B = 2E_\gamma/m_B$ is the rescaled photon energy. The parameter $\delta = 1-2E_\gamma^{\rm min}/m_B$ corresponds to the experimental lower cut on E_γ, and it has to satisfy $\delta > \Lambda_{\rm QCD}/m_B$; otherwise nonperturbative effects are not under control. It is straightforward to show that [15]

$$\overline{(1-x_B)}\Big|_{x_B>1-\delta} = \frac{\bar\Lambda}{m_B} + \left(1-\frac{\bar\Lambda}{m_B}\right)\int_{1-\delta}^{1} dx_b\,(1-x_b)\,\frac{1}{\Gamma_0}\,\frac{d\Gamma}{dx_b} - \frac{\bar\Lambda}{m_B}\,\delta(1-\delta)\,\frac{1}{\Gamma_0}\,\frac{d\Gamma}{dx_b}\bigg|_{x_b=1-\delta} + \ldots\,, \tag{7}$$

where $x_b = 2E_\gamma/m_b$, and $d\Gamma/dx_b$ is the photon spectrum in b quark decay. It has recently been computed away from the endpoint ($x_b=1$) to order $\alpha_s^2\beta_0$ [15]. $\Gamma_0 = G_F^2\,|V_{tb}V_{ts}^*|^2\,\alpha_{\rm em}\,C_7^2\,m_b^5/32\pi^4$ is the contribution of the tree level matrix element of $O_7(\sim \bar s_L\sigma_{\mu\nu}F^{\mu\nu}b_R)$ to the $\bar B\to X_s\gamma$ decay rate. All terms but the first one on the right-hand-side of Eq. (7) have perturbative expansions which begin at order α_s. The ellipses denote contributions of order $(\Lambda_{\rm QCD}/m_B)^3$, $\alpha_s(\Lambda_{\rm QCD}/m_B)^2$, and α_s^2 terms not enhanced by β_0, but it does not contain contributions of order $(\Lambda_{\rm QCD}/m_B)^2$ or additional terms of order $\alpha_s(\Lambda_{\rm QCD}/m_B)$. Terms in the operator product expansion proportional to $\lambda_{1,2}/m_b^2$ enter precisely in the form so that they are absorbed in m_B in Eq. (7) [14].

A determination of $\bar\Lambda$ is straightforward using Eq. (7). The left hand side is directly measurable, while the quantities entering on the right-hand-side are presented in Ref. [15]. The CLEO data in the region $E_\gamma > 2.1\,$GeV [19] yields the central values $\bar\Lambda_{\alpha_s^2\beta_0}\simeq 270\,$MeV and $\bar\Lambda_{\alpha_s}\simeq 390\,$MeV. Uncertainties due to the unknown order $\Lambda_{\rm QCD}^3/m_b^3$ terms in the

OPE give the short dashed ellipse in Fig. 2 [17], whose major axis is roughly perpendicular to those from $\bar{B} \to X_c e\bar{\nu}$. This is why it is important to determine $\bar{\Lambda}$ and λ_1 from both analyses. The potentially most serious uncertainty is from both nonperturbative and perturbative terms that are singular as $x_b \to 1$ and sum into a shape function [32] that modifies the spectrum near the endpoint. For sufficiently large δ these effects are not important. They have been estimated in Refs. [17,18] using phenomenological models. Whether these effects are small in a certain range of δ can be tested experimentally by checking if the extracted value of $\bar{\Lambda}$ is independent of δ. This would also improve our confidence that the total decay rate in the region $x_B > 1 - \delta$ can be predicted model independently.

There have been other important developments for the $\bar{B} \to X_s\gamma$ decay rate. The next-to-leading order computation of the total rate has been completed [33–36], reducing the theoretical uncertainties significantly. It was realized by Voloshin that beyond leading order there are terms in the OPE for the decay rate which are suppressed only by $\Lambda_{\rm QCD}^2/m_c^2$ instead of $\Lambda_{\rm QCD}^2/m_b^2$ [37]. In fact, there is a series of contributions of the form $(\Lambda_{\rm QCD}^2/m_c^2)\,(\Lambda_{\rm QCD} m_b/m_c^2)^n$, which gives for $n = 0$ a calculable correction of $\delta\Gamma/\Gamma = -(C_2/9C_7)(\lambda_2/m_c^2) \simeq 2.5\%$. It is also known that there are uncalculable contributions suppressed by α_s but not by $\Lambda_{\rm QCD}/m_b$ from photon coupling to light quarks, for which there is no OPE [38]. While no correction larger than a few percent has been identified, a better understanding of the nonperturbative contributions would be desirable once the four-quark operators are included.

III. $|V_{ub}|$ FROM THE $\bar{B} \to X_u e\bar{\nu}$ HADRON MASS SPECTRUM

The traditional method for extracting $|V_{ub}|$ involves a study of the electron energy spectrum in inclusive semileptonic B decay. Electrons with energies in the endpoint region $m_B/2 > E_e > (m_B^2 - m_D^2)/2m_B$ (in the B rest frame, and neglecting the pion mass) must arise from $b \to u$ transition. Since the size of this region is only about 300 MeV, at the present time it is not known how to make a model independent prediction for the spectrum in this region. Another possibility for extracting $|V_{ub}|$ is based on reconstructing the neutrino momentum. The idea is to reconstruct $p_{\bar{\nu}}$ and infer the invariant mass-squared of the hadronic final state, $s_H = (p_B - q)^2$, where $q = p_e + p_{\bar{\nu}}$. Semileptonic B decays satisfying $s_H < m_D^2$ must come from $b \to u$ transition [39–41]. The first analyses of LEP data utilizing this idea have been performed recently [42].

Both the invariant mass region, $s_H < m_D^2$, and the electron endpoint region, $m_B/2 > E_e > (m_B^2 - m_D^2)/2m_B$, receive contributions from hadronic final states with invariant masses between m_π and m_D. However, for the electron endpoint region the contribution of states with masses nearer to m_D is strongly suppressed kinematically. In fact, in the ISGW model [43] the electron endpoint region is dominated by the π and the ρ with higher mass states making a small contribution, and this region includes only of order 10% of the $\bar{B} \to X_u e\bar{\nu}$ rate. The situation is very different for the low invariant mass region, $s_H < m_D^2$. Now all states with invariant masses up to m_D contribute without any preferential weighting towards the lowest mass ones. In this case the ISGW model suggests the π and the ρ comprise only about a quarter of the B semileptonic decays to states with $s_H < m_D$, and only of order 10% of the $\bar{B} \to X_u e\bar{\nu}$ rate is excluded from this region. Consequently, it is much more likely that the first few terms in the OPE provide an accurate description of B semileptonic decay in the region $s_H < m_D^2$ than in the region $E_e > (m_B^2 - m_D^2)/2m_B$. Combining the invariant mass constraint with a modest cut on the electron energy will not destroy this conclusion.

Let us first consider the contribution of dimension three operators in the OPE to the hadronic mass-squared spectrum. This is equivalent to b quark decay, and implies a result for $d\Gamma/dE_0\,ds_0$ (where $E_0 = p_b \cdot (p_b - q)/m_b$ and $s_0 = (p_b - q)^2$ are the energy and invariant mass of the strongly interacting partons arising from the b quark decay) that is straightforward to calculate to order $\alpha_s^2\beta_0$ [39]. Even at this leading order in the OPE there are important nonperturbative effects that come from the difference between m_b and m_B. The most significant effect comes from $\bar{\Lambda}$, and including only it (*i.e.*, neglecting $\lambda_{1,2}$), the hadronic invariant mass s_H is related to s_0 and E_0 by [12]

$$s_H = s_0 + 2\bar{\Lambda}E_0 + \bar{\Lambda}^2 + \ldots \,. \qquad (8)$$

Changing variables from (s_0, E_0) to (s_H, E_0) and integrating E_0 over the range

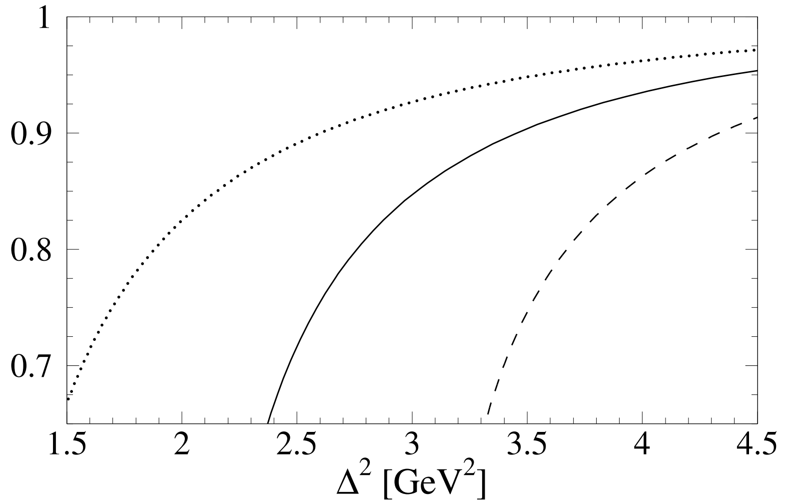

FIG. 3. Fraction of $\bar{B} \to X_u e\bar{\nu}$ decays with $s_H < \Delta^2$, for $\bar{\Lambda} = 0.2\,\mathrm{GeV}$ (dotted curve), $0.4\,\mathrm{GeV}$ (solid curve), and $0.6\,\mathrm{GeV}$ (dashed curve). Note that $m_D^2 = 3.5\,\mathrm{GeV}^2$. Nonperturbative effects not in $\bar{\Lambda}$ are neglected. (From Ref. [39].)

$$\sqrt{s_H} - \bar{\Lambda} < E_0 < \frac{1}{2m_B}\,(s_H - 2\bar{\Lambda}m_B + m_B^2), \tag{9}$$

gives $d\Gamma/ds_H$, where $\bar{\Lambda}^2 < s_H < m_B^2$. Feynman diagrams with only a u quark in the final state contribute at $s_0 = 0$, which corresponds to the region $\bar{\Lambda}^2 < s_H < \bar{\Lambda}m_B$.

Although $d\Gamma/ds_H$ is integrable in perturbation theory, it has a double logarithmic singularity at $s_H = \bar{\Lambda}m_B$. At higher orders in perturbation theory, increasing powers of $\alpha_s \ln^2[(s_H - \bar{\Lambda}m_B)/m_B^2]$ appear in the invariant mass spectrum. Therefore, $d\Gamma/ds_H$ in the vicinity of $s_H = \bar{\Lambda}m_B$ is hard to predict reliably even in perturbation theory. (In the region $s_H \lesssim \bar{\Lambda}m_B$ nonperturbative effects are also important.) The behavior of the spectrum near $s_H = \bar{\Lambda}m_B$ becomes less important for observables that average over larger regions of the spectrum, such as $d\Gamma/ds_H$ integrated over $s_H < \Delta^2$, with Δ^2 significantly greater than $\bar{\Lambda}m_B$. Fig. 3 shows the fraction of $\bar{B} \to X_u e\bar{\nu}$ events in the region $s_H < \Delta^2$ as a function of Δ^2 for three different values of $\bar{\Lambda}$. For a certain value of $\bar{\Lambda}$, Fig. 3 together with Eq. (13) can be used to extract $|V_{ub}|$ from data, up to the nonperturbative effects discussed next.

In the low mass region, $s_H \lesssim \bar{\Lambda}m_B$, nonperturbative corrections from higher dimension operators in the OPE are very important. Just as in the case of the electron endpoint region in semileptonic B decay or the photon energy endpoint region in radiative B decay, the most singular terms can be identified and summed into a shape function. These shape functions depend on the same infinite set of matrix elements. Since $\bar{\Lambda}m_b \approx 2\,\mathrm{GeV}^2$ is not too far from m_D^2, it is necessary to estimate the influence of the nonperturbative effects on the fraction of B decays with $s_H < m_D^2$. It is difficult to estimate this model independently, but upper bounds can be derived on the fraction of $\bar{B} \to X_u e\bar{\nu}$ events with $s_H > \Delta^2$ assuming that the shape function is positive [39]. In the ACCMM model [44] with reasonable parameters, the shape function causes a small (i.e., $\sim 4\%$ with $\bar{\Lambda} = 0.4\,\mathrm{GeV}$, and perturbative QCD corrections neglected) fraction of the events to have $s_H > m_D^2$ [39]. This suggests that sensitivity to unknown higher dimension operators in the OPE will probably not give rise to a large uncertainty in $|V_{ub}|$ if it is determined from the hadronic invariant mass spectrum in the region $s_H < m_D^2$. If experimental resolution forces one to consider a significantly smaller region, then the sensitivity to higher dimension operators increases rapidly.

In summary, to extract $|V_{ub}|$ from $d\Gamma/ds_H$ with small theoretical uncertainty, one needs to:

1. move the experimental cut Δ as close to m_D as possible;
2. determine $\bar{\Lambda}$ (at order α_s^2) with $\lesssim 50\,\mathrm{MeV}$ uncertainty.

Then a determination of $|V_{ub}|$ with $\sim 10\%$ theoretical uncertainty seems feasible.

IV. UPSILON EXPANSION

The main uncertainties in the theoretical predictions for inclusive B decay rates, e.g., $\bar{B} \to X_u e\bar{\nu}$, arise from the m_b^5 dependence on the b quark mass and the bad behavior of the series of perturbative corrections when it is written in terms of the pole mass. In fact, only the product of these quantities is unambiguous, but perturbative multi-loop calculations are most comfortably done in terms of the pole mass. Of course, one would like to eliminate the quark mass altogether from the predictions in favor of a physical observable. Here we present a new method of eliminating m_b in terms of the $\Upsilon(1S)$ meson mass [45] (instead of m_B and $\bar{\Lambda}$ discussed in Sec. II).

Let us consider the inclusive $\bar{B} \to X_u e\bar{\nu}$ decay rate [46]. At the scale $\mu = m_b$,

$$\Gamma(B \to X_u e\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3}\, m_b^5 \left[1 - 2.41\frac{\alpha_s}{\pi}\,\epsilon - 3.22\frac{\alpha_s^2}{\pi^2}\beta_0\,\epsilon^2 - 5.18\frac{\alpha_s^3}{\pi^3}\beta_0^2\,\epsilon^3 - \ldots - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \ldots\right]. \tag{10}$$

The variable $\epsilon \equiv 1$ denotes the order in the modified expansion. The complete order α_s^2 calculation was done recently [47], and the result is about 90% of the $\alpha_s^2\beta_0$ part. In comparison, the expansion of the $\Upsilon(1S)$ mass in terms of m_b [48] has a different structure,

$$\frac{m_\Upsilon}{2m_b} = 1 - \frac{(\alpha_s C_F)^2}{8}\left\{1\epsilon + \frac{\alpha_s}{\pi}\left[\left(\ell + \frac{11}{6}\right)\beta_0 - 4\right]\epsilon^2 + \left(\frac{\alpha_s\beta_0}{2\pi}\right)^2\left(3\ell^2 + 9\ell + 2\zeta(3) + \frac{\pi^2}{6} + \frac{77}{12}\right)\epsilon^3 + \ldots\right\}, \tag{11}$$

where $\ell = \ln[\mu/(m_b\alpha_s C_F)]$ and $C_F = 4/3$. In this expansion we assigned to each term one less power of ϵ than the power of α_s, because as we will sketch below, this is the consistent way of combining Eqs. (10) and (11). It is also convenient to choose the same renormalization scale μ. The prescription of counting $[\alpha_s(m_b)]^n$ in B decay rates as order ϵ^n, and $[\alpha_s(m_b)]^n$ in m_Υ as order ϵ^{n-1} is called the upsilon expansion. Note that it combines different orders in the α_s perturbation series in Eqs. (10) and (11).

The theoretical consistency of the upsilon expansion was shown at large orders for the terms containing the highest possible power of β_0, and to order ϵ^2 including non-Abelian contributions. An explicit calculation using the Borel transform of the static quark potential [49] shows that the coefficient of the order α_s^{n+2} term in Eq. (11) of the form $(\ell^n + \ell^{n-1} + \ldots + 1)$ exponentiates to give $\exp(\ell) = \mu/(m_b\alpha_s C_F)$, and corrects the mismatch of the power of α_s between the two series. This is also needed for the cancellation of the renormalon ambiguities in the energy levels as given by $2m_b$ plus the potential and kinetic energies [50,51]. The infrared sensitivity of Feynman diagrams can be studied by introducing a fictitious infrared cutoff λ. The infrared sensitive terms are nonanalytic in λ^2, such as $(\lambda^2)^{n/2}$ or $\lambda^{2n}\ln\lambda^2$, and arise from the low-momentum part of Feynman diagrams. Diagrams which are more infrared sensitive, i.e., have contributions $(\lambda^2)^{n/2}$ or $\lambda^{2n}\ln\lambda^2$ for small values of n, are expected to have larger nonperturbative contributions. Linear infrared sensitivity, i.e., terms of order $\sqrt{\lambda^2}$, are a signal of $\Lambda_{\rm QCD}$ effects, quadratic sensitivity, i.e., terms of order $\lambda^2\ln\lambda^2$ are a signal of $\Lambda_{\rm QCD}^2$ effects, etc. From Refs. [52] and [51] follows that the linear infrared sensitivity cancels in the upsilon expansion to order ϵ^2 (probably to all orders as well, but the demonstration of this appears highly non-trivial).

Substituting Eq. (11) into Eq. (10) and collecting terms of a given order in ϵ gives [45]

$$\Gamma(\bar{B} \to X_u e\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3}\left(\frac{m_\Upsilon}{2}\right)^5\left[1 - 0.115\epsilon - 0.035_{\rm BLM}\epsilon^2 - 0.005_{\rm BLM}\epsilon^3 - \frac{9\lambda_2 - \lambda_1}{2(m_\Upsilon/2)^2} + \ldots\right], \tag{12}$$

where the BLM [53] subscript indicates that only the corrections proportional to the highest power of β_0 have been kept. The complete order ϵ^2 term is $-0.041\epsilon^2$ [47]. The perturbation series, $1 - 0.115\epsilon - 0.035_{\rm BLM}\epsilon^2 - 0.005_{\rm BLM}\epsilon^3$, is far better behaved than the series in Eq. (10), $1 - 0.17\epsilon - 0.13_{\rm BLM}\epsilon^2 - 0.12_{\rm BLM}\epsilon^3$, or the series expressed in terms of the $\overline{\rm MS}$ mass, $1 + 0.30\epsilon + 0.19_{\rm BLM}\epsilon^2 + 0.05_{\rm BLM}\epsilon^3$. The uncertainty in the decay rate using Eq. (12) is much smaller than that in Eq. (10), both because the perturbation series is better behaved, and because m_Υ is better known (and better defined) than m_b. The relation between $|V_{ub}|$ and the total semileptonic $\bar{B} \to X_u e\bar{\nu}$ decay rate is [45]

$$|V_{ub}| = (3.06 \pm 0.08 \pm 0.08) \times 10^{-3}\left(\frac{\mathcal{B}(\bar{B} \to X_u e\bar{\nu})}{0.001}\,\frac{1.6\,{\rm ps}}{\tau_B}\right)^{1/2}, \tag{13}$$

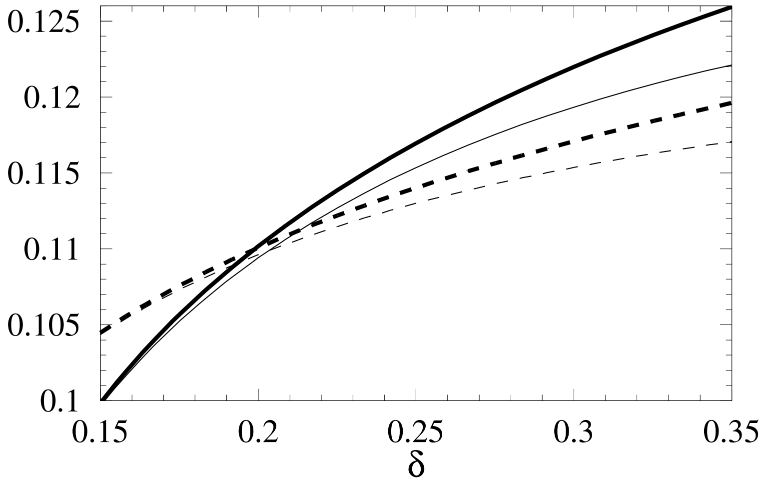

FIG. 4. Prediction in the upsilon expansion at order ϵ (thick dashed curve) and $(\epsilon^2)_{\rm BLM}$ (thick solid curve) for $\overline{(1-x_B)}|_{x_B>1-\delta}$ defined in Eq. (6). The thin curves show the O_7 contribution only. (From Ref. [15].)

The first error is obtained by assigning an uncertainty in Eq. (12) equal to the value of the ϵ^2 term and the second is from assuming a 100 MeV uncertainty in Eq. (11). The scale dependence of $|V_{ub}|$ due to varying μ in the range $m_b/2 < \mu < 2m_b$ is less than 1%. The uncertainty in λ_1 makes a negligible contribution to the total error. Of course, it is unlikely that $\mathcal{B}(\bar{B} \to X_u e\bar{\nu})$ will be measured without significant experimental cuts, for example, on the hadronic invariant mass (see Sec. III), but this method should reduce the uncertainties in such analyses as well.

The $\bar{B} \to X_c e\bar{\nu}$ decay depends on both m_b and m_c. It is convenient to express the decay rate in terms of m_Υ and λ_1 instead of m_b and m_c, using Eq. (11) and

$$m_b - m_c = \overline{m}_B - \overline{m}_D + \left(\frac{\lambda_1}{2\overline{m}_B} - \frac{\lambda_1}{2\overline{m}_D}\right) + \ldots, \tag{14}$$

where $\overline{m}_B = (3m_{B^*} + m_B)/4 = 5.313\,\text{GeV}$ and $\overline{m}_D = (3m_{D^*} + m_D)/4 = 1.973\,\text{GeV}$. We then find

$$\Gamma(\bar{B} \to X_c e\bar{\nu}) = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} \left(\frac{m_\Upsilon}{2}\right)^5 0.533 \times \left[1 - 0.096\epsilon - 0.029_{\rm BLM}\epsilon^2 - (0.28\lambda_2 + 0.12\lambda_1)/\text{GeV}^2\right], \tag{15}$$

where the phase space factor has also been expanded in ϵ. For comparison, the perturbation series in this relation when written in terms of the pole mass is $1 - 0.12\epsilon - 0.06\epsilon^2 - \ldots$ [46]. Equation (15) implies [45]

$$|V_{cb}| = (41.6 \pm 0.8 \pm 0.7 \pm 0.5) \times 10^{-3} \times \eta_{\rm QED} \left(\frac{\mathcal{B}(\bar{B} \to X_c e\bar{\nu})}{0.105} \, \frac{1.6\,\text{ps}}{\tau_B}\right)^{1/2}, \tag{16}$$

where $\eta_{\rm QED} \sim 1.007$ is the electromagnetic radiative correction. The uncertainties come from assuming an error in Eq. (15) equal to the ϵ^2 term, a $0.25\,\text{GeV}^2$ error in λ_1, and a 100 MeV error in Eq. (11), respectively. The second uncertainty can be removed by determining λ_1, as discussed in Sec. II. Other applications, such as for nonleptonic decays, exclusive semileptonic decays and $\bar{B} \to X_s\gamma$ photon spectrum were studied in Refs. [45,15].

The most important uncertainty in this approach is the size of nonperturbative contributions to m_Υ other than those which can be absorbed into the b quark mass. By dimensional analysis the size of this correction is of order $a^3\Lambda_{\rm QCD}^4$, where $a \sim 1/(m_b\alpha_s)$ is the Bohr radius of the Υ. Quantitative estimates, however, vary in a large range, and it is preferable to constrain such effects from data. The upsilon expansion yields parameter free predictions for $\overline{(1-x_B)}|_{x_B>1-\delta}$ defined in Eq. (6). The analog of Eq. (7) is [15]

$$\overline{(1-x_B)}\Big|_{x_B>1-\delta} = 1 - \frac{m_\Upsilon}{2m_B}\left[1 + 0.011\epsilon + 0.019(\epsilon^2)_{\rm BLM} - \langle 1 - x_b\rangle\Big|_{x_b>(2m_B/m_\Upsilon)(1-\delta)}\right], \tag{17}$$

For $E_\gamma > 2.1\,\mathrm{GeV}$ this relation gives 0.111, whereas the central value from the CLEO data is around 0.093. Fig. 4 shows the prediction for $\overline{(1-x_B)}|_{x_B>1-\delta}$ as a function of δ, both at order ϵ and $(\epsilon^2)_{\mathrm{BLM}}$. The perturbation expansion is very well behaved. In Eq. (17) nonperturbative contributions to m_Υ other than those which can be absorbed into the b quark mass have been neglected. If the nonperturbative contribution to Υ mass, Δ_Υ, were known, it could be included by replacing m_Υ by $m_\Upsilon - \Delta_\Upsilon$. For example, $\Delta_\Upsilon = +100\,\mathrm{MeV}$ increases $\overline{(1-x_B)}$ by 7%, so measuring $\overline{(1-x_B)}$ with such accuracy will have important implications for the physics of quarkonia as well as for B physics.

V. CONCLUSIONS

To conclude, let me emphasize the main points, and indicate what data would be important and useful in my opinion to address them:

- Experimental determination of $\bar\Lambda$ and λ_1 from the semileptonic $\bar B \to X_c e\bar\nu$ lepton energy and hadron mass spectra will reduce the theoretical uncertainties in $|V_{cb}|$ and $|V_{ub}|$.
  (Need: Double tagged lepton spectrum with smaller errors.)
- Photon energy spectrum in $\bar B \to X_s\gamma$ gives complimentary information on $\bar\Lambda$ and λ_1, even in the presence of an experimental cut on the photon energy.
  (Need: Spectrum with cut on E_γ lowered; even a few hundred MeV can reduce the uncertainties significantly.)
- To distinguish $\bar B \to X_u e\bar\nu$ from $\bar B \to X_c e\bar\nu$, cutting on the hadronic invariant mass is theoretically cleaner than cutting on the lepton energy.
  (Need: Experimental cut on hadron mass as close to m_D as possible, and a precise determination of $\bar\Lambda$.)
- The upsilon expansion is equivalent to using a short distance b quark mass, but it eliminates m_b altogether from the theoretical predictions in favor of m_Υ in a simple and consistent manner. It raises several interesting theoretical questions, and has many important applications.

ACKNOWLEDGMENTS

I thank Adam Falk and Alan Weinstein for inviting me to give this talk, and Christian Bauer for providing Fig. 2. Fermilab is operated by Universities Research Association, Inc., under DOE contract DE-AC02-76CH03000.

[1] K. Pitts (CDF Collaboration), Fermilab Joint Experimental Theoretical Physics Seminar, February 5, 1999.
[2] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B247 (1990) 399; M. Voloshin and M. Shifman, Sov. J. Nucl. Phys. 41 (1985) 120.
[3] I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B293 (1992) 430 [(E) Phys. Lett. B297 (1993) 477]; I.I. Bigi, M. Shifman, N.G. Uraltsev, and A. Vainshtein, Phys. Rev. Lett. 71 (1993) 496.
[4] A.V. Manohar and M.B. Wise, Phys. Rev. D49 (1994) 1310; B. Blok, L. Koyrakh, M. Shifman and A.I. Vainshtein, Phys. Rev. D49 (1994) 3356; T. Mannel, Nucl. Phys. B413 (1994) 396.
[5] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. 47 (1988) 511.
[6] C.G. Boyd, B. Grinstein, and A.V. Manohar, Phys. Rev. D54 (1996) 2081.
[7] I.I. Bigi, M.A. Shifman, N.G. Uraltsev, A.I. Vainshtein, Phys. Rev. D50 (1994) 2234; M. Beneke and V.M. Braun, Nucl. Phys. B426 (1994) 301.
[8] M. Beneke, V.M. Braun, and V.I. Zakharov, Phys. Rev. Lett. 73 (1994) 3058; M. Luke, A.V. Manohar, and M.J. Savage, Phys. Rev. D51 (1995) 4924; M. Neubert and C.T. Sachrajda, Nucl. Phys. B438 (1995) 235.
[9] M. Gremm, A. Kapustin, Z. Ligeti, and M.B. Wise, Phys. Rev. Lett. 77 (1996) 20; M. Gremm and I. Stewart, Phys. Rev. D55 (1997) 1226.
[10] M.B. Voloshin, Phys. Rev. D51 (1995) 4934.
[11] M. Gremm and A. Kapustin, Phys. Rev. D55 (1997) 6924.
[12] A.F. Falk, M. Luke, and M.J. Savage, Phys. Rev. D53 (1996) 2491; Phys. Rev. D53 (1996) 6316.

[13] A.F. Falk and M. Luke, Phys. Rev. D57 (1998) 424.
[14] A. Kapustin and Z. Ligeti, Phys. Lett. B355 (1995) 318.
[15] Z. Ligeti, M. Luke, A.V. Manohar, and M.B. Wise, FERMILAB-PUB-99-025-T [hep-ph/9903305].
[16] A.F. Falk, M. Luke, and M.J. Savage, Phys. Rev. D49 (1994) 3367.
[17] C. Bauer, Phys. Rev. D57 (1998) 5611.
[18] A.L. Kagan and M. Neubert, Eur. Phys. J. C7 (1999) 5.
[19] S. Glenn *et al.*, CLEO Collaboration, CLEO CONF 98-17.
[20] B.H. Smith and M.B. Voloshin, Phys. Lett. B340 (1994) 176.
[21] J. Bartelt *et al.*, CLEO Collaboration, CLEO CONF 98-21.
[22] R. Wang, Ph.D. Thesis, University of Minnesota (1994).
[23] See, e.g., The BaBar Physics Book (P.F. Harrison and H.R. Quinn, Eds.), Sec. 8.6.2, SLAC-R-504 (1998).
[24] N. Isgur and M.B. Wise, Phys. Lett. B232 (1989) 113; Phys. Lett. B237 (1990) 527.
[25] N. Isgur and M.B. Wise, Phys. Rev. Lett. 66 (1991) 1130.
[26] M. Zoeller, CLEO Collaboration, Talk presented at this Conference.
[27] ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. C73 (1997) 601; CLEO Collaboration, T.E. Browder *et al.*, Report no. CLEO CONF 96-2, ICHEP96 PA05-077.
[28] A.K. Leibovich, Z. Ligeti, I.W. Stewart, and M.B. Wise, Phys. Rev. Lett. 78 (1997) 3995; Phys. Rev. D57 (1998) 308; Z. Ligeti, hep-ph/9709396.
[29] N. Isgur and M.B. Wise, Phys. Rev. D43 (1991) 319.
[30] M.E. Luke, Phys. Lett. B252 (1990) 447.
[31] M. Neubert, Phys. Lett. B418 (1998) 173.
[32] M. Neubert, Phys. Rev. D49 (1994) 4623; D49 (1994) 3392; A.F. Falk, E. Jenkins, A.V. Manohar and M.B. Wise, Phys. Rev. D49 (1994) 4553; I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Int. J. Mod. Phys. A9 (1994) 2467; R.D. Dikeman, M. Shifman, and N.G. Uraltsev, Int. J. Mod. Phys. A11 (1996) 571.
[33] K. Chetyrkin, M. Misiak, and M. Munz, Phys. Lett. B400 (1997) 206; M. Misiak and M. Munz, Phys. Lett. B344 (1995) 308.
[34] K. Adel and Y.P. Yao, Phys. Rev. D49 (1994) 4945; C. Greub and T. Hurth, Phys. Rev. D56 (1997) 2934.
[35] A.J. Buras, M. Jamin, M.E. Lautenbacher, and P.H. Weisz, Nucl. Phys. B370 (1992) 69.
[36] C. Greub, T. Hurth, and D. Wyler, Phys. Rev. D54 (1996) 3350; Phys. Lett. B380 (1996) 385.
[37] M.B. Voloshin, Phys. Lett. B397 (1997) 275; Z. Ligeti, L. Randall, and M.B. Wise, Phys. Lett. B402 (1997) 178; A.K. Grant, A.G. Morgan, S. Nussinov, R.D. Peccei, Phys. Rev. D56 (1997) 3151; G. Buchalla, G. Isidori, S.J. Rey, Nucl. Phys. B511 (1998) 594.
[38] A. Kapustin, Z. Ligeti, and H.D. Politzer, Phys. Lett. B357 (1995) 653.
[39] A.F. Falk, Z. Ligeti, M.B. Wise, Phys. Lett. B406 (1997) 225.
[40] R.D. Dikeman and N.G. Uraltsev, Nucl. Phys. B509 (1998) 378; I. Bigi, R.D. Dikeman, and N. Uraltsev, Eur. Phys. J. C4 (1998) 453.
[41] V. Barger *et al.*, Phys. Lett. B251 (1990) 629; J. Dai, Phys. Lett. B333 (1994) 212;
[42] R. Barate *et al.*, ALEPH Collaboration, CERN EP/98-067; DELPHI Collaboration, contributed paper to the ICHEP98 Conference (Vancouver), paper 241; M. Acciarri *et al.*, L3 Collaboration, Phys. Lett. B436 (1998) 174.
[43] N. Isgur *et al.*, Phys. Rev. D39 (1989) 799; N. Isgur and D. Scora, Phys. Rev. D52 (1995) 2783.
[44] G. Altarelli *et al.*, Nucl. Phys. B208 (1982) 365; A. Ali and I. Pietarinen, Nucl. Phys. B154 (1979) 519.
[45] A.H. Hoang, Z. Ligeti, and A.V. Manohar, Phys. Rev. Lett. 82 (1999) 277; Phys. Rev. D59 (1999) 074017 [hep-ph/9811239].
[46] M. Luke, M.J. Savage, an M.B. Wise, Phys. Lett. B343 (1995) 329; B345 (1995) 301; P. Ball, M. Beneke, and V.M. Braun, Phys. Rev. D52 (1995) 3929.
[47] T. Ritbergen, TTP-99-11 [hep-ph/9903226].
[48] A. Pineda and F.J. Yndurain, UB-ECM-PF-97-34 [hep-ph/9711287]; K. Melnikov and A. Yelkhovskii, TTP-98-17 [hep-ph/9805270].
[49] U. Aglietti and Z. Ligeti, Phys. Lett. B364 (1995) 75.
[50] A.H. Hoang, M.C. Smith, T. Stelzer, and S. Willenbrock, UCSD-PTH-98-13 [hep-ph/9804227].
[51] M. Beneke, Phys. Lett. B434 (1998) 115.
[52] A. Sinkovics, R. Akhoury, and V.I. Zakharov, UM-TH-98-08 [hep-ph/9804401].
[53] S.J. Brodsky, G.P. Lepage, and P.B. Mackenzie, Phys. Rev. D28 (1983) 228.