Rescattering Effects in Heavy Quark Decays

Alexey A. Petrov
Department of Physics and Astronomy, Johns Hopkins University, Baltimore, Maryland 21218

We review some recent developments in the studies of direct CP violation and final state interactions in weak decays of heavy quarks.

I. INTRODUCTION AND MOTIVATION

Heavy quark decays serve as a powerful tool for testing the Standard Model and, since it probes the quarks of all three generations, provide invaluable possibilities to study CP violation. However, the interpretation of experimental observables in terms of fundamental parameters is often less than clear. Rare hadronic decays of B mesons, for example, proceed through both tree level Cabibbo-suppressed amplitudes and through one loop penguin amplitudes. On the one hand, this situation allows direct CP violating effects. On the other, these contributions complicate the extraction of Cabibbo-Kobayashi-Maskawa (CKM) angles and, in particular, the angle $\gamma \equiv \arg\left[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*\right]$.

The decay of a heavy hadron produces quarks in the final state. Because of their strong QCD interactions they continue to interact after the weak transition took place. Even after they have formed hadrons, there are still strong forces between them, and therefore, the problem of the *final state rescatterings or final state interactions (FSI)* is an important part of the physics of nonleptonic b-decays. The most pronounced effect of FSI is clearly in direct CP-violation where one compares the rates of a B-meson decay with the charged conjugated process. The corresponding asymmetries between the two decays depend on both a weak (CKM) and a strong rescattering phase provided by the FSI. Furthermore, a non-vanishing asymmetery requires two different final states produced by different weak amplitudes which can go into each other by a strong interaction rescattering. Thus, FSI directly affect the asymmetries and their size can be interpreted in terms of fundamental parameters *only* if these FSI phases are calculable.

As an example, let us first look at the recently measured combined branching ratios for $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ [1]. In the Standard Model, these decays are mediated by the $\Delta B = 1$ Hamiltonian, which takes the form

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\Big[V_{cb}V_{cs}^*\left(\sum_{i=1}^{2} C_i Q_i^{cs} + \sum_{i=3}^{6} C_i Q_i^{s} + \sum_{i=7}^{10} C_i Q_i^{s}\right) + V_{ub}V_{us}^*\left(\sum_{i=1}^{2} C_i Q_i^{us} + \sum_{i=3}^{6} C_i Q_i^{s} + \sum_{i=7}^{10} C_i Q_i^{s}\right)\Big] + \text{H.c.}, \quad (1)$$

The flavor structures of the current-current, QCD penguin, and electroweak penguin (EWP) operators are, respectively, $Q_{1,2}^{qs} \sim \bar{s}q\bar{q}b$, $Q_{3,..,6}^{s} \sim \bar{s}b\sum \bar{q}'q'$, and $Q_{7,..,10}^{s} \sim \bar{s}b\sum e_{q'}\bar{q}'q'$, the sum is over light quark flavors. The Wilson coefficients C_i are renormalization scale dependent, $C_{1,2}(1\ GeV) = \mathcal{O}(1)$, $C_{3,..,6,9}(1\ GeV) = \mathcal{O}(10^{-2})$, and $C_{7,8,10}(1\ GeV) \leq \mathcal{O}(10^{-3})$. Let us expand the decay amplitudes of interest according to their dependence on the elements of the CKM matrix,

$$\begin{aligned} A(B^+ \to \pi^+ K^0) &= A_{cs}^+ - A_{us}^+ e^{i\gamma} e^{i\delta_+}, \qquad & A(B^- \to \pi^- \bar{K}^0) &= A_{cs}^+ - A_{us}^+ e^{-i\gamma} e^{i\delta_+}, \\ A(B^0 \to \pi^- K^+) &= A_{cs}^0 - A_{us}^0 e^{i\gamma} e^{i\delta_0}, \qquad & A(\bar{B}^0 \to \pi^+ K^-) &= A_{cs}^0 - A_{us}^0 e^{-i\gamma} e^{i\delta_0}, \end{aligned} \quad (2)$$

where δ_0 and δ_+ are CP-conserving phases induced by the strong interaction. The first and second terms in each amplitude correspond to matrix elements of the first and second terms in $\mathcal{H}_{eff}$ (or their Hermitian conjugates), respectively. Note that each term is by itself scheme and renormalization scale independent.

One can asses the expected relative contributions of the operators in $\mathcal{H}_{\rm eff}$ to a given exclusive decay mode. The electroweak penguin operators are commonly neglected, since the contributions with a sizable Wilson coefficient, $C_9 Q_9^s$, are color suppressed or require rescattering from intermediate states. In this case isospin symmetry of the strong interactions leads to the simplification $A_{cs}^0 = A_{cs}^+$. It is now believed that the current-current operator contributions to $A_{cs}^{0,+}$ are roughly of same order as the QCD penguin operator contributions. The contribution of the current-current operators to A_{us}^0 is also expected to be of the same order, despite the CKM suppression, because of the large value of C_2, namely, $V_{ub}V_{us}^* C_2 \sim V_{cb}V_{cs}^* C_{3,..6}$. However, since for $B^\pm \to \pi^\pm K$ the relevant quark transition is $b \to d\bar{d}s$, one might expect the size of A^+_{us} relative to A^+_{cs} to be highly suppressed by the small ratio $|V_{ub}V^*_{us}/V_{cb}V^*_{cs}| \sim 0.02$. This would hold equally for the current-current and penguin operators. If, indeed, $r_+ = A^+_{us}/A^+_{cs} \sim |V_{ub}V^*_{us}/V_{cb}V^*_{cs}|$ is a good approximation, then there are two important consequences:

(i) Direct CP violation could be observed, in principle, through the CP asymmetry $\mathcal{A}^{\rm dir}_{\rm CP} \equiv \mathcal{A}^{\rm dir}_{\rm CP}(B^+ \to \pi^+ K^0)$,

$$\mathcal{A}^{\rm dir}_{\rm CP} = \frac{BR(B^+ \to \pi^+ K^0) - BR(B^- \to \pi^- \bar{K}^0)}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- \bar{K}^0)} = \frac{2r_+ \sin\gamma \sin\delta_+}{1 - 2r_+ \cos\gamma\cos\delta_+ + r_+^2} \,. \tag{3}$$

However, it would be small, $\mathcal{A}^{\rm dir}_{\rm CP}(B^+ \to \pi^+ K^0) \leq \mathcal{O}(\lambda^2)$, where $\lambda \simeq 0.22$ is the Wolfenstein parameter. “Hard” FSI estimates, where the u quarks in $Q^{us}_{1,2}$ are treated as a perturbative loop, give $\mathcal{A}^{\rm dir}_{\rm CP} \sim 1\%$.

(ii) Model-independent bounds could be obtained for the angle γ using only the *combined* branching ratios $BR(B^\pm \to \pi^\pm K)$ and $BR(B_d \to \pi^\mp K^\pm)$ [2,3]. One can construct the ratio

$$R = \frac{BR(B^0 \to \pi^- K^+) + BR(\bar{B}^0 \to \pi^+ K^-)}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- \overline{K^0})} = \left(\frac{A^0_{cs}}{A^+_{cs}}\right)^2 \frac{1 - 2r_0 \cos\gamma\cos\delta_0 + r_0^2}{1 - 2r_+ \cos\gamma\cos\delta_+ + r_+^2} \,, \tag{4}$$

where $r_0 = A^0_{us}/A^0_{cs}$. If A^+_{us} and EWP operator contributions are negligible, the ratio (4) takes the simple form

$$R = 1 - 2r_0 \cos\gamma\cos\delta_0 + r_0^2 \,. \tag{5}$$

The observable R may be minimized with respect to the parameter r_0, which (as $\cos^2\delta_0 \leq 1$) leads to the bound

$$\sin^2\gamma \leq R. \tag{6}$$

If true, a stringent bound on γ would be obtained if the reported by CLEO value $R_{\rm exp} = 1.00 \pm 0.40$ [4] turns out to be smaller than unity within experimental errors [5].

Rare B decays, like $B \to \pi K$, are suppressed in the Standard Model by either CKM matrix elements or small Wilson coefficients. Thus, these decays are potentially sensitive to New Physics. In the presence of New Physics, a large CP asymmetry can be induced, $\mathcal{A}^{\rm dir}_{\rm CP} \gg 1\%$ and R can be modified to violate the bound (6). The analysis leading to (6), however, explicitly assumes that the CKM angle γ does not enter the theoretical expression for the charged decay amplitudes (2), i.e. the absence of large contributions from the operators $Q^{us}_{1,2}$. This assumption is based on the observation that the quark level decay $b \to d\bar{d}s$ is not mediated direcly by $Q^{us}_{1,2}$. However, this treatment of the dynamics ignores the effects of soft rescattering effects at long distances, which can include the exchange of global quantum numbers such as charge and strangeness. In the absence of an argument that parton-hadron duality should hold in exclusive processes involving pions and kaons, one must conclude that the long distance physics of meson rescattering is not probed by the analysis of the final state rescattering based on perturbative QCD. In addition, the rescattering in question is *inelastic,* despite its quasi-elastic kinematics, and cannot be studied adequately in any model of purely *elastic* final state phases. Let us proceed with our example and investigate the impact of final state rescattering on the CP asymmetry $\mathcal{A}^{\rm dir}_{\rm CP}(B^\pm \to \pi^\pm K)$ and the ratio R. The rescattering process involves an intermediate on-shell state X, such that $B \to X \to K\pi$. In particular, we assume that there exists a generic (multibody) state $Kn\pi$. The charged and neutral channel amplitudes can then be written as

$$\begin{aligned} A(B^+ \to Kn\pi) &= A^{n+}_{cs} - A^{n+}_{us} e^{i\gamma} e^{i\delta^n_+} \,, \\ A(B^0 \to Kn\pi) &= A^{n0}_{cs} - A^{n0}_{us} e^{i\gamma} e^{i\delta^n_0} \,. \end{aligned} \tag{7}$$

Rescattering contributions, again decomposed according to their dependence on CKM factors, are given by

$$\begin{aligned} A(B^+ \to Kn\pi \to \pi^+ K^0) &= S^n_1 A^{n+}_{cs} - S^n_2 A^{n+}_{us} e^{i\gamma} \,, \\ A(B^0 \to Kn\pi \to \pi^- K^+) &= S^n_3 A^{n0}_{cs} - S^n_4 A^{n0}_{us} e^{i\gamma} \,, \end{aligned} \tag{8}$$

where S^n_i is the complex amplitude for rescattering from a given multibody final state to the channel of interest. In the limit of isospin symmetry $A^+_{cs} = A^0_{cs}$, and this equality is not spoiled by rescattering effects. The $i = 1,3,4$ rescattering amplitudes can be absorbed into the unknown amplitudes in Eq. (2).

Let us assume that the rescatterings of transitions mediated by $Q_{1,2}^{us}$ are significant enough to dominate A_{us}^{+}, so $A_{us}^{+}e^{i\delta_+} = \sum_n S_2^n A_{us}^{n+}$, and define $\epsilon = A_{us}^{+}/A_{cs}^{+}$. Let us also assume that rescattering effects do not dominate the overall decay, so we may retain just terms linear in ϵ. Then, $\mathcal{A}_{\rm CP}^{\rm dir}$ of Eq. (3) and R of Eq. (4) take the form

$$\mathcal{A}_{\rm CP}^{\rm dir} = \frac{2\epsilon \sin\gamma \sin\delta_+}{1-2\epsilon\cos\gamma\cos\delta_+}\,, \quad R = \frac{1-2r_0\cos\gamma\cos\delta_0 + r_0^2}{1-2\epsilon\cos\gamma\cos\delta_+}\,. \tag{9}$$

Once again, we may extremize R with respect to the unknown r_0,

$$R \geq \frac{1-\cos^2\gamma\cos^2\delta}{1-2\epsilon\cos\gamma\cos\delta_+}\,. \tag{10}$$

Using the same arguments as before with respect to the strong phases δ_0 and δ_+, we find the new bound

$$\sin^2\gamma \leq R(1+2\epsilon\sqrt{1-R}), \quad \text{or} \quad |\cos\gamma| \geq \sqrt{1-R} - \epsilon R\,. \tag{11}$$

It is clear that even a small rescattering amplitude $\epsilon \sim 0.1$ could induce a significant shift in the bound on γ deduced from R. For example, the fractional correction to the bound on $|\cos\gamma|$ is $\Delta \equiv \epsilon R/\sqrt{1-R}$. The value of Δ is a strong function of the experimentally observed $R_{\rm exp}$, $\Delta \simeq \epsilon$ for $R_{\rm exp} = 0.65$ and $\Delta \simeq 2\epsilon$ for $R_{\rm exp} = 0.80$ The bound deteriorates quickly as $R_{\rm exp} \to 1$. In addition, Eq. (9) implies that similar effect could in principle generate an $\mathcal{O}(10\%)$ CP asymmetry which is significantly larger than the bound $\mathcal{A}_{\rm CP}^{\rm dir}(B^\pm \to \pi^\pm K) \sim 1\%$. Therefore, in order to understand whether a large CP asymmetry signals New Physics, and whether it is possible to obtain a bound on γ, it is imperative to study FSI to obtain an order of magnitude estimate of the effect.

II. FINAL STATE RESCATTERING

Final state interactions arise as a consequence of the unitarity of the $\mathcal{S}$-matrix, $\mathcal{S}^\dagger\mathcal{S} = 1$, and involve the rescattering of physical particles in the final state. The $\mathcal{T}$-matrix, defined by $\mathcal{S} = 1 + i\mathcal{T}$, obeys the optical theorem:

$$\mathcal{D}isc\ \mathcal{T}_{B\to f} \equiv \frac{1}{2i}\left[\langle f|\mathcal{T}|B\rangle - \langle f|\mathcal{T}^\dagger|B\rangle\right] = \frac{1}{2}\sum_i \langle f|\mathcal{T}^\dagger|i\rangle\langle i|\mathcal{T}|B\rangle \quad , \tag{12}$$

where $\mathcal{D}isc$ denotes essentially the imaginary part. Using CPT in the form $\langle\bar{f}|\mathcal{T}|\bar{B}\rangle^* = \langle\bar{B}|\mathcal{T}^\dagger|\bar{f}\rangle = \langle f|\mathcal{T}^\dagger|B\rangle$ this can be tranformed into the more intuititve form

$$\langle\bar{f}|\mathcal{T}|\bar{B}\rangle^* = \sum_i \langle f|\mathcal{S}^\dagger|i\rangle\langle i|\mathcal{T}|B\rangle \quad . \tag{13}$$

Here, the states $|i\rangle$ represent all possible final states (including $|f\rangle$ itself) which can be reached from the state $|B\rangle$ by the weak transition matrix $\mathcal{T}$. The right hand side of Eq. (13) can then be viewed as a weak decay of $|B\rangle$ into $|i\rangle$ followed by a strong rescattering of $|i\rangle$ into $|f\rangle$. Thus, we identify $\langle f|\mathcal{S}^\dagger|i\rangle$ as a CP-conserving FSI rescattering of particles. Notice that if $|i\rangle$ is an eigenstate of $\mathcal{S}$ with a phase $e^{2i\delta}$, we have

$$\langle\bar{i}|\mathcal{T}|\bar{B}\rangle^* = e^{-2i\delta_i}\langle i|\mathcal{T}|B\rangle \quad . \tag{14}$$

which implies equal rates for the charge conjugated decays and hence no CP asymmtery. Therefore, at least two different states with equal quantum numbers must exist which can be connected by strong rescattering. Also

$$\langle\bar{i}|\mathcal{T}|\bar{B}\rangle = e^{i\delta}T_i\langle i|\mathcal{T}|B\rangle = e^{i\delta}T_i^* \tag{15}$$

The matrix elements T_i are assumed to be the "bare" decay amplitudes, calculated e.g. in factorization approximation [6] and have no rescattering phases. This implies that these transition matrix elements between charge conjugated states are just the complex conjugated ones of each other. Eq. (15) is known as Watson's theorem [7].

The above considerations allow to formulate a condition for the non-vanishing CP asymmetry. It requires two different final states which undergo strong transitions into each other. The strong phase is then nothing but the occurence of the *physical* intermediate state $|B_\beta\rangle$ and arises when summing over the intermediate states.

The final state rescatterings of high energy particles may be divided into 'soft' and 'hard' scattering. Soft scattering occurs primarily in the forward direction with limited transverse momentum, having a distribution which falls exponentially with a scale of order 0.5 GeV. Soft scattering might be best described by hadronic states. At higher transverse momentum one encounters the region of hard scattering, which falls only as a power of the transverse momentum. Collisions involving hard scattering are interpreted as interactions between the quarks and gluons of QCD. Note that it is possible to generate FSI phases in nonleptonic B-decays into charmless final states in perturbative QCD [8–10]: there are two ways to reach a given final state, via the tree diagram $b \to u\bar{u}s$, and via $b \to c\bar{c}s$ process, with subsequent final state rescattering of the two charmed quarks into two up quarks (penguin diagram). Since the energy release in b-decay is of the order $m_b > 2m_c$, the rescattered c-quarks can go on-shell generating CP conserving phase and thus $\mathcal{A}_{CP}^{dir}$.

For the soft FSI, the low energy effective theory of strong interactions can be used to reliably estimate FSI phase differences in the kaon system. In the D system final state rescattering has been studied assuming the dominance of intermediate resonances [11]. In the B system, where the density of the resonances available is large due to the increased energy, a different approach must be employed. One can use the fact that the $b-$quark mass is large compared to the QCD scale. Then, the leading order behavior of soft final state phases in the $m_b \to \infty$ limit can be investigated. This can be done by considering first the *elastic* channel, and demonstrating that elastic rescattering does not disappear in the limit of large m_B. Since the unitarity of the $\mathcal{S}$-matrix requires that the inelastic channels are indeed the dominant contributors to soft rescattering, such contributions have to share a similar behavior in the heavy quark limit. The elastic channel is convenient because of the optical theorem which connects the forward (imaginary) invariant amplitude $\mathcal{M}$ to the total cross section,

$$\mathcal{I}m\ \mathcal{M}_{f\to f}(s,\ t=0) = 2k\sqrt{s}\sigma_{f\to \text{all}} \sim s\sigma_{f\to \text{all}} \quad , \tag{16}$$

where s is the squared center-of-mass energy and t is the squared momentum transfer. The asymptotic total cross sections are known experimentally to increase slowly with energy and can be parameterized by the form [12,13]:

$$\sigma(s) = X\left(\frac{s}{s_0}\right)^{0.08} + Y\left(\frac{s}{s_0}\right)^{-0.56} \quad , \tag{17}$$

where $s_0 = \mathcal{O}(1)$ GeV is a typical hadronic scale. Thus, the imaginary part of the forward elastic scattering amplitude (16) increases asymptotically as $s^{1.08}$. Considering only the imaginary part of the amplitude, and building in the known exponential fall-off of the elastic cross section in t ($t < 0$) [14] by writing

$$i\mathcal{I}m\ \mathcal{M}_{f\to f}(s,t) \simeq i\beta_0\left(\frac{s}{s_0}\right)^{1.08} e^{bt} \quad , \tag{18}$$

one can calculate the contribution of the imaginary part of the elastic amplitude to the unitarity relation for a final state $f = a + b$ with kinematics $p'_a + p'_b = p_a + p_b$ and $s = (p_a + p_b)^2$:

$$\begin{aligned} \mathcal{D}isc\ \mathcal{M}_{B\to f} &= \frac{1}{2}\int \frac{d^3p'_a}{(2\pi)^3 2E'_a}\frac{d^3p'_b}{(2\pi)^3 2E'_b}(2\pi)^4\delta^{(4)}(p_B - p'_a - p'_b)\Big(-i\beta_0\left(\frac{s}{s_0}\right)^{1.08} e^{b(p_a-p'_a)^2}\Big)\mathcal{M}_{B\to f} \\ &= -\frac{1}{16\pi}\frac{i\beta_0}{s_0 b}\left(\frac{m_B^2}{s_0}\right)^{0.08}\mathcal{M}_{B\to f} \quad , \end{aligned} \tag{19}$$

where $t = (p_a - p'_a)^2 \simeq -s(1-\cos\theta)/2$, and $s = m_B^2$.

On can refine the argument further, since the phenomenology of high energy scattering is well accounted for by the Regge theory [14,15]. In the Regge model, scattering amplitudes are described by the exchanges of Regge trajectories (families of particles of differing spin) with the leading contribution given by the Pomeron exchange. Calculating the Pomeron contribution to the elastic final state rescattering in $B \to \pi\pi$ one finds [16]

$$\mathcal{D}isc\ \mathcal{M}_{B\to\pi\pi}|_{\rm Pomeron} = -i\epsilon\mathcal{M}_{B\to\pi\pi}, \qquad \epsilon \simeq 0.21 \ . \tag{20}$$

It is important that the Pomeron-exchange amplitude is seen to be almost purely imaginary. However, of chief significance is the identified weak dependence of ϵ on m_B – the $(m_B^2)^{0.08}$ factor in the numerator is attenuated by the $\ln(m_B^2/s_0)$ dependence in the effective value of b.

The analysis of the elastic channel suggests that, at high energies, FSI phases are *mainly generated by inelastic effects.* This conclusion immediately follows from the fact that the high energy cross section is mostly inelastic. This also follows from the fact that the Pomeron elastic amplitude is almost purely imaginary. Since the study of elastic rescattering has yielded a $\mathcal{T}$-matrix element $\mathcal{T}_{ab\to ab} = 2i\epsilon$, i.e. $\mathcal{S}_{ab\to ab} = 1 - 2\epsilon$, and since the constraint of unitarity of the $\mathcal{S}$-matrix implies that the off-diagonal elements are $\mathcal{O}(\sqrt{\epsilon})$, with ϵ approximately $\mathcal{O}(m_B^0)$ in powers of m_B and numerically $\epsilon < 1$, then the inelastic amplitude must also be $\mathcal{O}(m_B^0)$ and of magnitude $\sqrt{\epsilon} > \epsilon$. Similar conclusions follow from the consideration of the final state unitarity relations.

The very presence of inelastic effects suggests a physical picture similar and complimentary to the "color transparency argument". This argument suggests that a "small" (compared to the typical hardonic size $1/\Lambda_{QCD}$) color-singlet two-quark configuration does not interact with the soft gluonic environment. Thus, if the two-body decay is dominated by this particular quark configuration with all other quark configurations yielding multiparticle final states, one might expect that the effects of FSI are suppressed in the decays of ultra-heavy particles. However, this quark configuration is realized only on the edge of the available phase space. Therfore, in the limit $m_b \to \infty$ one ultimately encounters the situation where the quarks easily combine to form a multiparticle state which then undergoes rescattering into the two-body final state. Analysis of the final-state unitarity relations in their general form,

$$\mathcal{D}isc\ \mathcal{M}_{B\to f_1} = \frac{1}{2}\sum_k \ \mathcal{M}_{B\to k}T^\dagger_{k\to f_1} \ \ , \tag{21}$$

is complicated due to the many contributing intermediate states present at the B mass. However, it is possible to illustrate the systematics of inelastic scattering in a simple two-channel model. This example involves a two-body final state f_1 undergoing elastic scattering and a final state f_2 which represents 'everything else'. We assume that the elastic amplitude is purely imaginary. Thus, the scattering can be described in the one-parameter form

$$S = \left(\begin{array}{cc} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{array}\right) \ , \qquad T = \left(\begin{array}{cc} 2i\sin^2\theta & \sin 2\theta \\ \sin 2\theta & 2i\sin^2\theta \end{array}\right) \ , \tag{22}$$

where, from our elastic-rescattering calculation, we identify $\sin^2\theta \equiv \epsilon$. The unitarity relations become

$$\begin{aligned} \mathcal{D}isc\ \mathcal{M}_{B\to f_1} &= -i\sin^2\theta\mathcal{M}_{B\to f_1} + \frac{1}{2}\sin 2\theta\mathcal{M}_{B\to f_2} \ \ , \\ \mathcal{D}isc\ \mathcal{M}_{B\to f_2} &= \frac{1}{2}\sin 2\theta\mathcal{M}_{B\to f_1} - i\sin^2\theta\mathcal{M}_{B\to f_2} \end{aligned} \tag{23}$$

Denoting $\mathcal{M}_1^0$ and $\mathcal{M}_2^0$ to be the decay amplitudes in the limit $\theta \to 0$, an exact solution to Eq. (23) is given by

$$\mathcal{M}_{B\to f_1} = \cos\theta\mathcal{M}_1^0 + i\sin\theta\mathcal{M}_2^0 \ , \qquad \mathcal{M}_{B\to f_2} = \cos\theta\mathcal{M}_2^0 + i\sin\theta\mathcal{M}_1^0 \ \ . \tag{24}$$

In this example we see that the phase is given by the inelastic scattering with a result of order

$$\frac{\mathcal{I}m\ \mathcal{M}_{B\to f}}{\mathcal{R}e\ \mathcal{M}_{B\to f}} \sim \sqrt{\epsilon}\,\frac{\mathcal{M}_2^0}{\mathcal{M}_1^0} \ \ . \tag{25}$$

Clearly, for physical B decay, we no longer have a simple one-parameter $\mathcal{S}$ matrix ,and, with many channels, cancellations or enhancements are possible for the sum of many contributions. However, the main feature of the above result is expected to remain – that inelastic channels cannot vanish because they are required to make the discontinuity real and that the phase is systematically of order $\sqrt{\epsilon}$ from these channels and thus does not vanish in the large m_B limit. Moreover, it is possible to show that inelastic FSI can contribute to CP violating asymmetries at the leading order in m_B [16]. Non-zero FSI phases have been recently observed by CLEO [17].

We should note that radiative weak decays of B mesons are also affected by FSI. For instance, the extraction of V_{td} from $B \to \rho\gamma$ is hampered by uncertainties related to certain long distance effects from the on-shell hadron rescattering with subsequent conversion of one of the hadrons into the photon. This contribution could be sizeable [18].

III. TESTING THE MODELS OF FINAL STATE INTERACTIONS

(i) Model Independent Bounds on the FSI Corrections. In view of the large theoretical uncertainties involved in the calculation of the FSI contributions, it would be extremely useful to find a phenomenological method by which to bound the magnitude of the FSI contribution. The observation of a larger asymmetry would then be a signal for New Physics. Here the application of flavor $SU(3)$ flavor symmetry, provides powerful methods to obtain a direct upper bound on the FSI contribution.

The simplest example involves bounding FSI in $B \to \pi K$ decays using $B^\pm \to K^\pm K$ transitions. The effective Hamiltonian for $b \to d$ decays may be obtained from (1) by the substitution $s \to d$. In analogy with Eq. (2) the amplitudes may be decomposed according to their dependence on CKM factors, giving

$$
\begin{aligned}
A(B^+ \to K^+ \bar{K}^0) &= A_{cd} - A_{ud} e^{i\gamma} e^{i\delta}\,, \\
A(B^- \to K^- K^0) &= A_{cd} - A_{ud} e^{-i\gamma} e^{i\delta}\,.
\end{aligned}
\tag{26}
$$

Invariance under the $SU(3)$ rotation $\exp(i\frac{\pi}{2}\lambda_7)$, i.e., interchange of s and d quark fields, implies equalities among operator matrix elements and amplitudes,

$$
\begin{aligned}
\langle K^- K^0 | Q_i^{qd} | B^- \rangle &= \langle K^0 \pi^- | Q_i^{qs} | B^- \rangle, \quad q = u, c, ; \quad i = 1, 2 \\
\langle K^- K^0 | Q_i^{d} | B^- \rangle &= \langle K^0 \pi^- | Q_i^{s} | B^- \rangle, \quad i = 3, .., 10\,. \\
A_{ud} e^{i\delta} &= A_{us}^{+} e^{i\delta_+} \frac{V_{ud}}{V_{us}} (1 + R_{ud})\,, \quad A_{cd} = A_{cs}^{+} \frac{V_{cd}}{V_{cs}} (1 + R_{cd}),
\end{aligned}
\tag{27}
$$

where R_{ud} and R_{cd} parameterize $SU(3)$ violation, which is typically of the order of $20 - 30\%$. Note that it is only an $SU(2)$ subgroup of $SU(3)$, namely U-spin, which is required to derive these relations. Since the B^- carries $U = 0$ and the transition operators Q_i^{qd} and Q_i^d carry $U = \frac{1}{2}$, it is only the $U = \frac{1}{2}$ component of the $K^- K^0$ final state which couples to the decay channel. As a result, an upper bound on ϵ follows from the ratio

$$
R_K = \frac{BR(B^+ \to K^+ \overline{K^0}) + BR(B^- \to K^- K^0)}{BR(B^+ \to K^0 \pi^+) + BR(B^- \to \overline{K^0} \pi^-)}\,.
\tag{28}
$$

After some algebra, we obtain for ϵ and $\mathcal{A}_{\rm CP}^{\rm dir}$

$$
\begin{aligned}
\epsilon &< \lambda \sqrt{R_K}\,(1 + {\rm Re}[R_{ud}]) + \lambda^2 (R_K + 1) \cos\gamma \cos\delta_+ + \mathcal{O}(\lambda^3, \lambda^2 R_{ud,cd})\,, \\
|\mathcal{A}_{\rm CP}^{\rm dir}| &< 2\lambda \sqrt{R_K R}\,(1 + Re[R_{ud}]) + 2\lambda^2 \sqrt{R}\,(R_K \sqrt{1 - R} + R_K + 1) + \mathcal{O}(\lambda^3, \lambda^2 R_{ud,cd})\,.
\end{aligned}
\tag{29}
$$

Using the experimental bound $R_K < 1.9$ one obtains $\epsilon < 0.4$ and $\mathcal{A}_{\rm CP}^{\rm dir} < 0.6$. More interesting constraints on ϵ and $\mathcal{A}_{\rm CP}^{\rm dir}$ could also be be obtained [19].

(ii) Direct Observation. As we have shown above, the amplitude for the decay of a B-meson into some final state f includes a direct contribution $A(B \to f)$ and a sum over the contributions $A(B \to i \to f)$, which corresponds to the weak decays of the B-meson into intermediate hadronic states i, followed by the strong scattering of i into f. The possibility of significant final state scattering effects becomes real when considering rare decays, for which the amplitude $A(B \to f)$ is suppressed compared to $A(B \to i)$. In the ideal case, when the direct contribution is absent, one may be able to isolate the effect of FSI completely. While this situation might not be realized in the nature, rare weak decays offer tantalizing possibility of the *direct observation* of the effects of FSI.

One of the possibilities involve dynamically suppressed decays which proceed via weak annihillation diagrams. It has been argued that final state interactions can modify the decay amplitudes, violating the expected hierarchy of amplitues. For example, it is expected that the amplitudes that do not involve spectator quarks (such as tree-level or penguin amplitudes) dominate over the diagrams involving spectator quarks (e.g. weak annihilation or weak rescattering amplitudes). In many cases, large amplitudes might contribute to the processes involving spectator quarks through the final state rescattering [20,21]. It must be stressed that although the predictive power of available methods is limited and most of the estimates are based on the two-body rescattering diagrams, some conclusions can still be reached. For instance, it is possible to show [21] that the rescattering from the dominant channel leads to the suppression only of the order $\lambda \sim 0.2$ compared to $f_B/m_B \sim \lambda^2$ obtained from the naive quark diagram estimate.

Another class of decays involve the so-called OZI-violating modes, i.e. those which cannot be realized in terms of quark diagrams without annihillation of at least one pair of the quarks. It includes modes like $\overline{B}_d^0 \to \phi\phi, D^0\phi$ and $J/\psi\phi$. For instance, the direct contribution to $\overline{B}_d^0 \to \phi\phi$ involves a suppressed space-like penguin diagram. However, the unitarity of the $\mathcal{S}$-matrix, $\mathcal{S}^\dagger \mathcal{S} = 1$, implies that this decay can also proceed via the OZI-allowed weak transition followed by final state rescattering into the final state under consideration. These two-step OZI violating processes were intensively studied in connection with certain low-energy processes [22,23]. In B-decays these OZI-allowed steps involve multiparticle intermediate states and might provide a source for significant violation of the OZI rule. For instance, the FSI contribution can proceed via $\overline{B}_d^0 \to \eta^{(\prime)}\eta^{(\prime)} \to \phi\phi$, $\overline{B}_d^0 \to D^{*0}\eta^{(\prime)} \to D^0\phi$ and $\overline{B}_d^0 \to \psi'\eta^{(\prime)} \to J/\psi\phi$. The intermediate state might also include an additional set of pions. The weak decay into the intermediate state occurs at tree level, through the $(u\overline{u} + d\overline{d})/\sqrt{2}$ component of the $\eta^{(\prime)}$ wavefunction, whereas the strong scattering into the final state involves the $s\bar{s}$ component [24]. Hence the possibility of using these decay modes as direct probes of the FSI contributions to B decay amplitudes. It is however possible to show that there exist strong cancellations [23,25] among various *two body* intermediate channels. In the example of $\overline{B}_d^0 \to \phi\phi$, the cancellation among η and η' is almost complete, so the effect is of the second order in the $SU(3)$-breaking corrections

$$Disc\ \mathcal{M}_{B\to\phi\phi} = O(\delta^2, \Delta^2, \delta\Delta) f_\eta F_0 A, \quad \delta = f_{\eta'} - f_\eta, \ \Delta = F_0' - F_0, \tag{30}$$

with $A \sim s^{\alpha_0 - 1} e^{i\pi\alpha_0/2}/8b$. This implies that the OZI-suppressed decays provide an excellent probe of the multiparticle FSI. Given the very clear signature, these decay modes could be probed at the upcoming B-factories.

[1] A.F. Falk, A.L. Kagan, Y. Nir and A.A. Petrov, Phys. Rev. **D57**, 4290 (1998); M. Neubert, Phys. Lett. **B424**, 152 (1998); D. Atwood and A. Soni, Phys. Rev. **D58**, 036005 (1998).
[2] R. Fleischer and T. Mannel, Phys. Rev. **D57**, 2752 (1998).
[3] Y. Grossman *et. al.* Nucl. Phys. **B511**, 69 (1998).
[4] CLEO collaboration, Phys. Rev. Lett. **80**, 3456 (1998).
[5] M. Gronau and J.L. Rosner, Phys. Rev. **D57**, 6843 (1998).
[6] For example, see J.F. Donoghue and A.A. Petrov, Phys. Lett. **B 393**, 149 (1997) and references therein.
[7] K.M. Watson, Phys. Rev. **88**, 1163 (1952).
[8] L. Wolfenstein, Phys. Rev. **D43**, 151 (1991).
[9] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).
[10] J.-M. Gerard and W.-S. Hou, Phys. Rev. D **43**, 2909 (1991); Yu. Dokshitser, N. Uraltsev, JETP Lett. **52** (10), 1109 (1990); L.-L. Chau, H.-Y. Cheng, Phys. Rev. Lett. **59**, 958 (1987); A. Ali, G. Kramer and C.D. Lu, Phys. Rev. **D59**, 014005 (1999).
[11] F. Buccella, A. Pugliese and M. Lusignoli, Phys. Lett. **B 379**, 249 (1996).
[12] Particle Data Group, L. Montanet *et al.*, Phys. Rev. **D50**, 1173 (1994).
[13] A. Donnachie and P.V. Landshoff, Phys. Lett. **B296**, 227 (1992).
[14] P.D.B. Collins, *Introduction to Regge Theory*, (Cambridge University Press, Cambridge, England 1977).
[15] N.G. Deshpande and C.O. Dib, Phys. Lett. **B319** 313 (1993); H. Zheng, Phys. Lett. **B356** 107 (1995).
[16] J.F. Donoghue, E. Golowich, A.A. Petrov, and J.M. Soares, Phys. Rev. Lett. **77**, 2178 (1996).
[17] J. Rodriguez (CLEO), Talk given at "Heavy Quarks 98", Preprint CLEO TALK98-22.
[18] J.F. Donoghue, E. Golowich, and A.A. Petrov, Phys. Rev. **D55**, 2657 (1997).
[19] A. Buras and R. Fleischer, hep-ph/9810260; R. Fleischer, Eur. Phys. J. **C6**, 451 (1999); M. Suzuki, Phys. Rev. **D58**, 111504 (1998); M. Gronau and D. Pirjol, hep-ph/9902482.
[20] B. Blok, I. Halperin, Phys. Lett. **B385**, 324 (1996).
[21] B. Blok, M. Gronau, and J.L. Rosner, Phys. Rev. Lett. **78**, 3999 (1997).
[22] P. Geiger and N. Isgur, Phys. Rev. Lett. **67**, 1066 (1991).
[23] H. J. Lipkin, Nucl. Phys. **B291**, 720 (1987).
[24] J. F. Donoghue, Phys. Rev. **D33**, 1516 (1986).
[25] J.F. Donoghue, E. Golowich, A.A. Petrov, and J.M. Soares, unpublished.