Isospin Violation in B $\to \pi\pi$ Decays

S. Gardner
Department of Physics and Astronomy, University of Kentucky,
Lexington, KY 40506-0055
E-mail: gardner@pa.uky.edu

An isospin analysis of $B \to \pi\pi$ decays yields $\sin 2\alpha$, where α is the usual CKM angle $\alpha \equiv \arg[-V_{td}V_{tb}^*/(V_{ud}V_{ub}^*)]$ without hadronic uncertainty if isospin is a perfect symmetry. Yet isospin symmetry is broken not only by electroweak effects but also by the u and d quark mass difference — the latter drives $\pi^0 - \eta, \eta'$ mixing and converts the isospin-perfect triangle relation between the $B \to \pi\pi$ amplitudes to a quadrilateral. The error incurred in $\sin 2\alpha$ through the neglect of the resulting isospin-violating effects can be significant, particularly if $\sin 2\alpha$ is small.

I. INTRODUCTION

In the standard model, CP violation is characterized by a single phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, rendering its elements complex. The CKM matrix is also unitary, so that determining whether or not this is empirically so is a central test of the standard model's veracity [1]. Ascertaining whether the angles of the unitarity triangle, α, β, and γ, empirically sum to π and whether its angles are compatible with the measured lengths of its sides lie at the heart of these tests of the standard model.

We study the impact of isospin violation on the extraction of $\sin 2\alpha$ from an isospin analysis in $B \to \pi\pi$ decays [2]. Isospin is broken not only by electroweak effects but also by the u and d quark mass difference. The latter drives $\pi^0 - \eta, \eta'$ mixing [3], which, in turn, generates an amplitude in $B \to \pi\pi$ not included in the isospin analysis. Thus, although the effect of electroweak penguins is estimated to be small [4–6], when all the effects of isospin violation are included, the error in the extracted value of $\sin 2\alpha$ can be significant [7].

To review the isospin analysis in $B \to \pi\pi$ decays, due to Gronau and London [2], let us consider the time-dependent asymmetry $A(t)$ [8]:

$$A(t) = \frac{(1 - |r_{f_{CP}}|^2)}{(1 + |r_{f_{CP}}|^2)} \cos(\Delta m\, t) - \frac{2(\mathrm{Im}\, r_{f_{CP}})}{(1 + |r_{f_{CP}}|^2)} \sin(\Delta m\, t) \;, \tag{1}$$

where $r_{f_{CP}} = (V_{tb}^* V_{td}/V_{tb}V_{td}^*)(\overline{A}_{f_{CP}}/A_{f_{CP}}) \equiv e^{-2i\phi_m}\overline{A}_{f_{CP}}/A_{f_{CP}}$, $A_{f_{CP}} \equiv A(B_d^0 \to f_{CP})$, and $\Delta m \equiv B_H - B_L$ [9].

Denoting the amplitudes $B^+ \to \pi^+\pi^0$, $B^0 \to \pi^0\pi^0$, and $B^0 \to \pi^+\pi^-$ by A^{+0}, A^{00}, and A^{+-}, respectively, and introducing A_I to denote an amplitude of final-state isospin I, we have [2]

$$\frac{1}{2}A^{+-} = A_2 - A_0 \;\; ; \;\; A^{00} = 2A_2 + A_0 \;\; ; \;\; \frac{1}{\sqrt{2}}A^{+0} = 3A_2 \;, \tag{2}$$

where analogous relations exist for A^{-0}, $\overline{A}^{00}$, and $\overline{A}^{+-}$ in terms of $\overline{A}_2$ and $\overline{A}_0$. If isospin were perfect, then the Bose symmetry of the $J = 0$ $\pi\pi$ state would permit amplitudes merely of $I = 0, 2$, so that the amplitudes $B^\pm \to \pi^\pm\pi^0$ would be purely $I = 2$. In this limit the penguin contributions are strictly of $\Delta I = 1/2$ character, so that they cannot contribute to the $I = 2$ amplitude: no CP violation is possible in the $\pi^\pm\pi^0$ final states. The penguin contribution in $B^0 \to \pi^+\pi^-$, or in $\bar{B}^0 \to \pi^+\pi^-$, can then be isolated and removed by determining the relative magnitude and phase of the $I = 0$ to $I = 2$ amplitudes. We have

$$r_{\pi^+\pi^-} = e^{-2i\phi_m}\frac{(\overline{A}_2 - \overline{A}_0)}{(A_2 - A_0)} = e^{2i\alpha}\frac{(1 - \overline{z})}{(1 - z)} \;, \tag{3}$$

where $z(\overline{z}) \equiv A_0/A_2(\overline{A}_0/\overline{A}_2)$ and $\overline{A}_2/A_2 \equiv \exp(-2i\phi_t)$ with $\phi_t \equiv \arg(V_{ud}V_{ub}^*)$ and $\phi_m + \phi_t = \beta + \gamma = \pi - \alpha$ in the standard model [8]. Given $|A^{+-}|$, $|A^{00}|$, $|A^{+0}|$, and their charge conjugates, the measurement of $\mathrm{Im}\, r_{\pi^+\pi^-}$ determines $\sin 2\alpha$, modulo discrete ambiguities in $\arg((1 - \overline{z})/(1 - z))$, which correspond geometrically to the orientation of the "triangle" of amplitudes associated with Eq. (2), namely

$$A^{+-} + 2A^{00} = \sqrt{2}A^{+0} \ , \tag{4}$$

with respect to $|A^{+0}| = |A^{-0}|$ and that of its charge conjugate. The triangles' relative orientation can be resolved via a measurement of $\mathrm{Im}\, r_{\pi^0\pi^0}$ as well [2], and thus $\sin 2\alpha$ is determined uniquely.

II. ISOSPIN VIOLATION AND π^0-η, η' MIXING

We examine the manner in which isospin-violating effects impact the extraction of $\sin 2\alpha$, for isospin is merely an approximate symmetry. The charge difference between the u and d quarks engenders a $\Delta I = 3/2$ electroweak penguin contribution, which is outside the scope of the delineated isospin analysis [2], although methods have been suggested to include them [10,11]. This is the only manner in which the u-d charge difference enters our analysis, so that we term this source of isospin breaking an "electroweak effect." The u-d quark mass difference can also engender a $\Delta I = 3/2$ strong penguin contribution through isospin-breaking in the hadronic matrix elements. Moreover, strong-interaction isospin violation drives $\pi^0 - \eta, \eta'$ mixing [3], admitting an $I = 1$ amplitude. Although electroweak penguin contributions are estimated to be small [4–6], other isospin-violating effects, such as π^0-η, η' mixing, can be important [7,12].

To include the effects of π^0-η, η' mixing, we write the pion mass eigenstate $|\pi^0\rangle$ in terms of the $\mathrm{SU}(3)_f$-perfect states $|\phi_3\rangle$, $|\phi_8\rangle$, and $|\phi_0\rangle$, where, in the quark model, $|\phi_3\rangle = |u\overline{u} - d\overline{d}\rangle/\sqrt{2}$, $|\phi_8\rangle = |u\overline{u} + d\overline{d} - 2s\overline{s}\rangle/\sqrt{6}$, and $|\phi_0\rangle = |u\overline{u} + d\overline{d} + s\overline{s}\rangle/\sqrt{3}$. Explicit relations between the physical and $\mathrm{SU}(3)_f$-perfect states can be realized by expanding QCD to leading order in $1/N_c$, momenta, and quark masses to yield a low-energy, effective Lagrangian in which the pseudoscalar meson octet and singlet states are treated on the same footing [3,13]. Diagonalizing the quadratic terms in ϕ_3, ϕ_8, and ϕ_0 of the resulting effective Lagrangian determines the mass eigenstates π^0, η, and η' and yields, to leading order in isospin violation [3],

$$|\pi^0\rangle = |\phi_3\rangle + \varepsilon(\cos\theta|\phi_8\rangle - \sin\theta|\phi_0\rangle) + \varepsilon'(\sin\theta|\phi_8\rangle + \cos\theta|\phi_0\rangle) \ , \tag{5}$$

where $\cos\theta|\phi_8\rangle - \sin\theta|\phi_0\rangle = |\eta\rangle + O(\varepsilon)$, and $\sin\theta|\phi_8\rangle + \cos\theta|\phi_0\rangle = |\eta'\rangle + O(\varepsilon')$. Moreover, $\varepsilon = \epsilon_0\chi\cos\theta$ and $\varepsilon' = -2\epsilon_0\tilde{\chi}\sin\theta$, with $\chi = 1 + (4m_K^2 - 3m_\eta^2 - m_\pi^2)/(m_\eta^2 - m_\pi^2) \approx 1.23$, $\tilde{\chi} = 1/\chi$, $\epsilon_0 \equiv \sqrt{3}(m_d - m_u)/(4(m_s - \hat{m}))$, and $\hat{m} \equiv (m_u + m_d)/2$ [3]. Thus the magnitude of isospin breaking is controlled by the SU(3)-breaking parameter $m_s - \hat{m}$. The η-η' mixing angle θ is found to be $\sin 2\theta = -(4\sqrt{2}/3)(m_K^2 - m_\pi^2)/(m_{\eta'}^2 - m_\eta^2)$ so that $\theta \approx -22^\circ$ [3]. The resulting $\varepsilon = 1.14\epsilon_0$ is comparable to the one-loop-order chiral perturbation theory result of $\varepsilon = 1.23\epsilon_0$ in $\eta \to \pi^+\pi^-\pi^0$ [14,3]. Empirical constraints also exist on the *sign* of π^0-η, η' mixing. That is, the ratio of the reduced matrix elements in K_{l3} decays, namely, $K^+ \to \pi^0 e^+ \nu_e$ and $K_L^0 \to \pi^- e^+ \nu_e$, is given by [15]

$$\left(\frac{f_+^{K^+\pi^0}}{f_+^{K_L^0\pi^-}} \right)^{\mathrm{expt}} = 1.029 \pm 0.010 \ . \tag{6}$$

Using the Lagrangian of Ref. [3] and the quark masses $m_q(\mu = 1 \text{ GeV})$ of Ref. [16] yields

$$\left(\frac{f_+^{K^+\pi^0}}{f_+^{K_L^0\pi^-}} \right) = 1 + \sqrt{3}\varepsilon_8 \approx 1.018 \pm 0.010 \ , \tag{7}$$

where ε_8 is the $\phi_3 - \phi_8$ mixing angle, $\varepsilon_8 \equiv \varepsilon\cos\theta + \varepsilon'\sin\theta$. Note, for comparison, that the one-loop-order chiral perturbation theory result is 1.022 [17]. In regard to the $\sin 2\alpha$ results to follow, it is worth noting that the isospin-violating parameters we have adopted appear conservative with respect to the existing experimental constraints. Using $m_q(\mu = 2.5\,\mathrm{GeV})$ of Ali *et al.* [16], we find $\varepsilon = 1.4 \cdot 10^{-2}$ and $\varepsilon' = 7.7 \cdot 10^{-3}$; we use these values in the subsequent calculations.

In the presence of π^0-η, η' mixing, the $B \to \pi\pi$ amplitudes become

$$A^{-0} = \langle \pi^-\phi_3|\mathcal{H}^{\mathrm{eff}}|B^-\rangle + \varepsilon_8\langle \pi^-\phi_8|\mathcal{H}^{\mathrm{eff}}|B^-\rangle + \varepsilon_0\langle \pi^-\phi_0|\mathcal{H}^{\mathrm{eff}}|B^-\rangle \tag{8}$$

$$\overline{A}^{00} = \langle \phi_3\phi_3|\mathcal{H}^{\mathrm{eff}}|\bar{B}^0\rangle + \varepsilon_8\langle \phi_3\phi_8|\mathcal{H}^{\mathrm{eff}}|\bar{B}^0\rangle + \varepsilon_0\langle \phi_3\phi_0|\mathcal{H}^{\mathrm{eff}}|\bar{B}^0\rangle \ , \tag{9}$$

where ε_0 is the $\phi_3 - \phi_0$ mixing angle, $\varepsilon_0 \equiv \varepsilon' \cos\theta - \varepsilon \sin\theta$. Note that either of the π^0 mesons in the $B^0 \to \pi^0\pi^0$ amplitude can suffer π^0-η, η' mixing; the factor of two associated with this appears as $2\overline{A}^{00}$ in Eq. (10). The $B \to \pi\pi$ amplitudes satisfy

$$\overline{A}^{+-} + 2\overline{A}^{00} - \sqrt{2}\,A^{-0} = 2\varepsilon_8\langle\phi_3\phi_8|\mathcal{H}^{\rm eff}|\bar{B}^0\rangle + 2\varepsilon_0\langle\phi_3\phi_0|\mathcal{H}^{\rm eff}|\bar{B}^0\rangle - \sqrt{2}\varepsilon_8\langle\pi^-\phi_8|\mathcal{H}^{\rm eff}|B^-\rangle - \sqrt{2}\varepsilon_0\langle\pi^-\phi_0|\mathcal{H}^{\rm eff}|B^-\rangle \,, \tag{10}$$

and thus the triangle relation of Eq.(4) becomes a quadrilateral. We ignore the relatively unimportant mass differences $m_{\pi^\pm} - m_{\pi^0}$ and $m_{B^\pm} - m_{B^0}$.

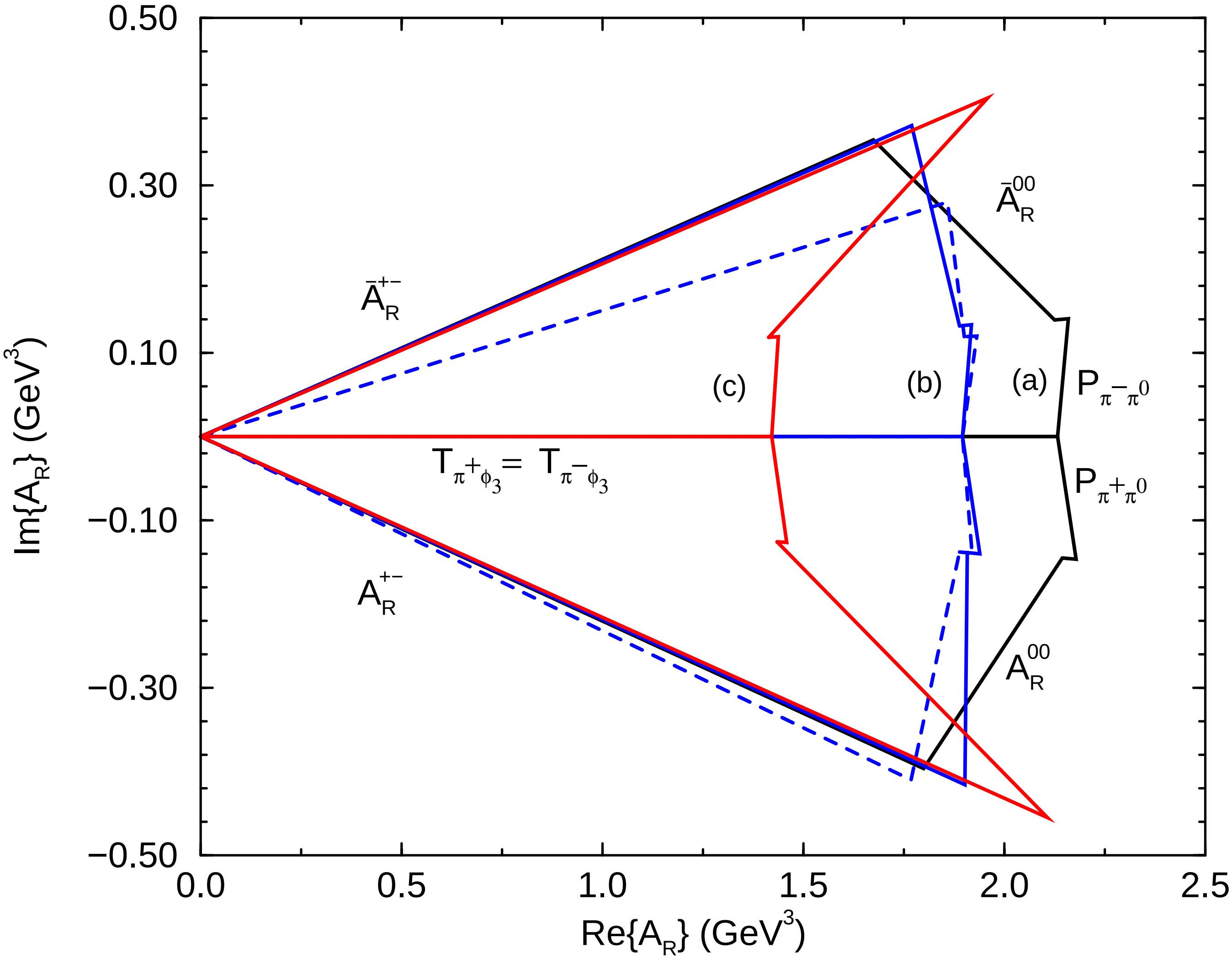

FIG. 1. Reduced amplitudes in $B \to \pi\pi$ in the factorization approximation with $[N_c,\, k^2/m_b^2]$ for a) [2,0.5], b) [3, 0.5] (solid line) and [3, 0.3] (dashed line), and c) [∞, 0.5]. Note that $\overline{A}_R^{00} \equiv 2\overline{A}^{00}/((G_F/\sqrt{2})iV_{ub}V_{ud}^*)$, $\overline{A}_R^{+-} \equiv \overline{A}^{+-}/((G_F/\sqrt{2})iV_{ub}V_{ud}^*)$, and $A_R^{-0} \equiv \sqrt{2}A^{-0}/((G_F/\sqrt{2})iV_{ub}V_{ud}^*)$. The charged modes are separated into tree and penguin contributions, so that $A_R^{+0} \equiv T_{\pi^+\phi_3} + P_{\pi^+\pi^0}$ and $A_R^{-0} \equiv T_{\pi^-\phi_3} + P_{\pi^-\pi^0}$, where $P_{\pi^\pm\pi^0}$ includes the isospin-violating tree contribution in $A_R^{\pm 0}$ as well. The shortest side in each polygon is the vector defined by the RHS of Eq. (10); it is non-zero only in the presence of π^0-η, η' mixing.

III. RESULTS

We proceed by computing the individual amplitudes using the $\Delta B = 1$ effective Hamiltonian resulting from the operator product expansion in QCD in next-to-leading logarithmic (NLL) order [16], using the factorization approximation for the hadronic matrix elements. In this context, we can then apply the isospin analysis delineated above to infer $\sin 2\alpha$ and thus estimate its theoretical systematic error, incurred through the neglect of isospin violating effects. The effective Hamiltonian $\mathcal{H}^{\rm eff}$ for $b \to dq\bar{q}$ decay can be parametrized as [16]

$$\mathcal{H}^{\rm eff} = \frac{G_F}{\sqrt{2}} \left[V_{ub}V_{ud}^*(C_1 O_1^u + C_2 O_2^u) + V_{cb}V_{cd}^*(C_1 O_1^c + C_2 O_2^c) - V_{tb}V_{td}^* \left(\sum_{i=3}^{10} C_i O_i + C_g O_g \right) \right] , \tag{11}$$

where O_i and O_g are as per Ref. [16]; we also adopt their Wilson coefficients C_i and C_g, computed in the naive dimensional regularization scheme at a renormalization scale of $\mu = 2.5$ GeV [16]. In NLL order, the Wilson coefficients are scheme-dependent; yet, after computing the hadronic matrix elements to one-loop-order, the matrix elements of the effective Hamiltonian are still scheme-independent [18]. This can be explicitly realized through the replacement $\langle dq\bar{q}|\mathcal{H}^{\rm eff}|b\rangle = (G_F/\sqrt{2})\langle dq\bar{q}|[V_{ub}V_{ud}^*(C_1^{\rm eff}O_1^u + C_2^{\rm eff}O_2^u) - V_{tb}V_{td}^*\sum_{i=3}^{10} C_i^{\rm eff} O_i]|b\rangle^{\rm tree}$, where "tree" denotes a tree-level matrix element and the $C_i^{\rm eff}$ are from Ref. [16]. The $C_i^{\rm eff}$ are complex [19] and depend on both the CKM matrix parameters and k^2, where k is the momentum transferred to the $q\bar{q}$ pair in $b \to dq\bar{q}$ decay. Noting Ref. [20] we use $\rho = 0.12$, $\eta = 0.34$, and $\lambda = 0.2205$ [16,21] unless otherwise stated. One expects $m_b^2/4 \lesssim k^2 \lesssim m_b^2/2$ [22]; we use $k^2/m_b^2 = 0.3, 0.5$ in what follows. We define the decay constants $\langle \pi^-(p)|\bar{d}\gamma_\mu\gamma_5 u|0\rangle \equiv -if_\pi p_\mu$ and $\langle \phi_i(p)|\bar{u}\gamma_\mu\gamma_5 u|0\rangle \equiv -if_{\phi_I}^u p_\mu$, and use the flavor content of the SU(3)$_f$-perfect states to relate $f_{\phi_i}^q$ to f_π. Finally, using the quark equations of motion with PCAC and introducing $a_i \equiv C_i^{\rm eff} + C_{i+1}^{\rm eff}/N_c$ for i odd and $a_i \equiv C_i^{\rm eff} + C_{i-1}^{\rm eff}/N_c$ for i even, the $B^- \to \pi^-\phi_3$ matrix element in the factorization approximation with use of the Fierz relations is

$$\begin{aligned} \langle\ \pi^-\phi_3|\mathcal{H}^{\rm eff}|B^-\rangle &= \frac{G_F}{\sqrt{2}}[V_{ub}V_{ud}^*(if_\pi F_{B^-\phi_3}(m_{\pi^-}^2)a_1 + if_{\phi_3}^u F_{B^-\pi}(m_{\pi^0}^2)a_2) - V_{tb}V_{td}^* \\ &\times (if_\pi F_{B^-\phi_3}(m_{\pi^-}^2)(a_4 + a_{10} + \frac{2m_{\pi^-}^2(a_6+a_8)}{(m_u+m_d)(m_b-m_u)}) - if_{\phi_3}^u F_{B^-\pi}(m_{\pi^0}^2) \\ &\times (a_4 + \frac{3}{2}(a_7 - a_9) - \frac{1}{2}a_{10} + \frac{m_{\pi^0}^2(a_6 - \frac{1}{2}a_8)}{m_d(m_b - m_d)}))] \,. \end{aligned} \tag{12}$$

The transition form factors are given by $F_{B^-\pi}(q^2) = (m_{B^-}^2 - m_{\pi^-}^2)F_0^{B\to\pi}(0)/(1-q^2/M_{0^+}^2)$, where we use $F_0^{B\to\pi}(0) = 0.33$ and $M_{0^+} = 5.73$ GeV as per Refs. [16,23]. Also $F_{B\phi_3} = F_{B\pi}/\sqrt{2}$, $F_{B\phi_8} = F_{B\pi}/\sqrt{6}$, and $F_{B\phi_0} = F_{B\pi}/\sqrt{3}$. Note that the a_4 and a_6 terms, which are associated with the strong penguin operators, only contribute to the $\langle\pi^-\phi_3|\mathcal{H}^{\rm eff}|B^-\rangle$ matrix element if $m_u \neq m_d$ or if $f_{\phi_3}^u F_{B^-\pi}(m_{\pi^0}^2) \neq f_\pi F_{B^-\phi_3}(m_{\pi^-}^2)$ — we neglect this latter contribution as we set $m_{\pi^\pm} = m_{\pi^0}$. The strong-penguin contributions, which are isospin-violating, explicitly realize the induced $\Delta I = 3/2$ effect discussed previously, for the amplitude $\langle\pi^-\phi_3|\mathcal{H}^{\rm eff}|B^-\rangle$, in concert with the amplitudes $\langle\pi^-\pi^+|\mathcal{H}^{\rm eff}|\bar{B}^0\rangle$ and $\langle\phi_3\phi_3|\mathcal{H}^{\rm eff}|\bar{B}^0\rangle$, satisfy the triangle relation of Eq. (4). Thus the $\langle\pi^-\phi_3|\mathcal{H}^{\rm eff}|B^-\rangle$ amplitude

TABLE I. Strong phases and inferred values of $\sin 2\alpha$ [2] from amplitudes in the factorization approximation with N_c and $k^2/m_b^2 = 0.5$. The strong phase $2\delta_{\rm true}$ is the opening angle between the $\overline{A}_R{}^{+-}$ and $A_R{}^{+-}$ amplitudes in Fig. 1, whereas $2\delta_{\rm GL}$ is the strong phase associated with the closest matching $\sin 2\alpha$ values, denoted $(\sin 2\alpha)_{\rm GL}$, from Im $r_{\pi^+\pi^-}$/Im $r_{\pi^0\pi^0}$, respectively. The bounds $|2\delta_{\rm GQI}|$ and $|2\delta_{\rm GQII}|$ on $2\delta_{\rm true}$ from Eqs. (2.12) and (2.15) of Ref. [24] are also shown. All angles are in degrees. We input a) $\sin 2\alpha = 0.0432$ [16,21], b) $\sin 2\alpha = -0.233$ ($\rho = 0.2, \eta = 0.35$) [25], and c) $\sin 2\alpha = 0.959$ ($\rho = -0.12, \eta = 0.34$).

case	N_c	$2\delta_{\rm true}$	$\lvert 2\delta_{\rm GQI}\rvert$	$\lvert 2\delta_{\rm GQII}\rvert$	$\lvert 2\delta_{\rm GL}\rvert$	$(\sin 2\alpha)_{\rm GL}$
a	2	24.4	24.5	12.5	13.4	-0.145/0.153
a	3	24.2	16.1	15.3	15.4	-0.107/0.133
a	∞	23.8	55.8	19.4	17.7	-0.0595/0.101
b	2	19.6	22.0	8.3	9.4	-0.399/-0.0213
b	3	19.4	12.9	12.3	12.4	-0.349/-0.0599
b	∞	19.2	56.2	17.3	15.7	-0.287/-0.104
c	2	28.3	34.4	14.8	4.9	0.769/0.701 (∗, †)
c	3	28.0	22.8	17.4	4.0	0.662/0.692 (∗, †)
c	∞	27.6	42.7	21.2	19.5	0.912/0.967

∗ The matching procedure fails to choose a $\sin 2\alpha$ which is as close to the input value as possible.

† The discrete ambiguity in the strong phase is resolved wrongly.

can still be deemed purely $I = 2$ even if $m_u \neq m_d$, as in Eq. (12). In the presence of π^0-η, η' mixing, however, the $B^- \to \pi^- \pi^0$ amplitude can no longer be purely $I = 2$, as the RHS of Eq. (10) is non-zero and Eq. (4) is no longer satisfied.

Numerical results for the reduced amplitudes A_R and $\overline{A}_R$, where $\overline{A}_R^{00} \equiv 2\overline{A}^{00}/((G_F/\sqrt{2}) i V_{ub} V_{ud}^*)$, $\overline{A}_R^{+-} \equiv \overline{A}^{+-}/((G_F/\sqrt{2}) i V_{ub} V_{ud}^*)$, and $A_R^{-0} \equiv \sqrt{2} A^{-0}/((G_F/\sqrt{2}) i V_{ub} V_{ud}^*)$, with $N_c = 2, 3, \infty$ and $k^2/m_b^2 = 0.3, 0.5$ are shown in Fig.1. A_R^{+0} and A_R^{-0} are broken into tree and penguin contributions, so that $A_R^{+0} \equiv T_{\pi^+ \phi_3} + P_{\pi^+ \pi^0}$ and $A_R^{-0} \equiv T_{\pi^- \phi_3} + P_{\pi^- \pi^0}$. Note that "$P_{\pi^\pm \pi^0}$" is defined to include the isospin-violating tree contribution in $A_R^{\pm 0}$ as well. The shortest side in each polygon is the vector defined by the RHS of Eq. (10). The values of $\sin 2\alpha$ extracted from the computed amplitudes with N_c — note that N_c is regarded as an effective parameter in this context — and $k^2/m_b^2 = 0.5$ are shown in Table I; the results for $k^2/m_b^2 = 0.3$ are similar and have been omitted. For reference, the ratio of penguin to tree amplitudes in $B^- \to \pi^- \pi^0$ is $|P|/|T| \sim (2.2 - 2.7)\% |V_{tb} V_{td}^*|/|V_{ub} V_{ud}^*|$ for $N_c = 2, 3$ and k^2 as above. Were electroweak penguins the only source of isospin violation, then $|P|/|T| \sim (1.4 - 1.5)\% |V_{tb} V_{td}^*|/|V_{ub} V_{ud}^*|$, commensurate with the estimate of 1.6% in Ref. [4].

In the presence of π^0-η, η' mixing, the $\overline{A}_R^{+-}$, $\overline{A}_R^{-0}$, and $\overline{A}_R^{00}$ amplitudes obey a quadrilateral relation as per Eq. (10). Consequently, the values of $\sin 2\alpha$ extracted from Im $r_{\pi^+\pi^-}$ and Im $r_{\pi^0\pi^0}$ measurements can not only differ markedly from the value of $\sin 2\alpha$ input but also need not match. The incurred error in $\sin 2\alpha$ increases as the value to be extracted decreases; the structure of Eq. (3) suggests this, for as $\sin 2\alpha$ decreases, the quantity Im $((1 - \overline{z})/(1 - z))$ becomes more important to determining the extracted value. It is useful to constrast the impact of the various isospin-violating effects. The presence of $\Delta I = 3/2$ penguin contributions, be they from $m_u \neq m_d$ or electroweak effects, shift the extracted value of $\sin 2\alpha$ from its input value, yet the "matching" of the $\sin 2\alpha$ values from the Im $r_{\pi^+\pi^-}$ and Im $r_{\pi^0\pi^0}$ determinations is unaffected. This arises as the amplitudes in question still satisfy the triangle relations implied by Eq. (4). The mismatch troubles seen in Table I are driven by π^0-η, η' mixing, though the latter shifts the values of $\sin 2\alpha$ extracted from Im $r_{\pi^+\pi^-}$ as well. Picking the closest matching values of $\sin 2\alpha$ in the two final states also picks the solutions closest to the input value; the exceptions are noted in Table I. The matching procedure can also yield the wrong strong phase; in case c) of Table I with $N_c = 2, 3$, the triangles of the chosen solutions "point" in the same direction, whereas they actually point oppositely. If $|A^{00}|$ and $|\overline{A}^{00}|$ are small [2] the complete isospin analysis may not be possible, so that we also examine the utility of the bounds recently proposed by Grossman and Quinn [24] on the strong phase $2\delta_{\rm true} \equiv \arg((1 - \overline{z})/(1 - z))$ of Eq. (3). The bounds $2\delta_{\rm GQI}$ and $2\delta_{\rm GQII}$ given by their Eqs. (2.12) and (2.15) [24], respectively, follow from Eq. (4), and thus can be broken by isospin-violating effects. As shown in Table I, the bounds typically are broken, and their efficacy does not improve as the value of $\sin 2\alpha$ to be reconstructed grows large.

To conclude, we have considered the role of isospin violation in $B \to \pi\pi$ decays and have found the effects to be significant. Most particularly, the utility of the isospin analysis in determining $\sin 2\alpha$ strongly depends on the value to be reconstructed. The error in $\sin 2\alpha$ from a Im $r_{\pi^+\pi^-}$ measurement grows markedly larger as $\sin 2\alpha$ grows small — this is the region of $\sin 2\alpha$ currently favored, albeit weakly, by phenomenology [16,21,25,26]. The effects found arise in part because the penguin contribution in $B \to \pi^+\pi^-$, e.g., is itself small; we estimate $|P|/|T| < 9\% |V_{tb} V_{td}^*|/|V_{ub} V_{ud}^*|$. Relative to this scale, the impact of π^0-η, η' mixing is significant. This is displayed in another way in Table II. The "penguin pollution" in $B \to \pi^+\pi^-$ is such that were no isospin analysis applied, the error in α would be of the order of $10^\circ - 20^\circ$. The isospin-violating effects in $B \to \pi^+\pi^-$ suggest that the error in α is still of the order of 5° after the

TABLE II. Errors in α were Im $r_{\pi^+\pi^-}$ taken to be $\sin 2\alpha$ ($|\delta\alpha|_{\rm raw}$) and from applying the Gronau-London analysis [2] in the presence of isospin-violating corrections ($|\delta\alpha|_{\rm GL}$) for amplitudes computed in the factorization approximation with $N_c = 2$ and $k^2/m_b^2 = 0.5$. All angles are in degrees. Cases a), b), and c) are defined as in Table I.

case	N_c	Im $r_{\pi^+\pi^-}$	$\lvert\delta\alpha\rvert_{\rm raw}$	$\lvert\delta\alpha\rvert_{\rm GL}$
a	2	-0.346	11.3	5.4
b	2	-0.514	8.8	5.0
c	2	0.642	16.8	11.6 (*)

* The discrete ambiguity in the strong phase is resolved wrongly in this case — see Table I.

Gronau-London [2] analysis is applied. Yet, were the penguin contributions in $B \to \pi\pi$ larger, pressing the need for the corrections of the isospin analysis, the isospin-violating effects considered would still be germane, for not only would the $\Delta I = 3/2$ penguin contributions likely be larger, but the $B \to \pi\eta$ and $B \to \pi\eta'$ contributions could also be larger as well [27]. To conclude, we have shown that the presence of π^0-η, η' mixing breaks the triangle relationship, Eq. (4), usually assumed [2] and can mask the true value of $\sin 2\alpha$.

ACKNOWLEDGEMENTS

I would like to thank Kam-Biu Luk and German Valencia for the opportunity to speak at this meeting. This work was supported by the U.S. Department of Energy under DE-FG02-96ER40989.

REFERENCES

[1] R. Fleischer, Int. J. Mod. Phys. A **12**, 2459 (1997).
[2] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990). Note $A_{\rm GL}^{+-} = A^{+-}/\sqrt{2}$ and $A_{\rm GL}^{\pm 0} = A^{\pm 0}/\sqrt{2}$.
[3] H. Leutwyler, Phys. Lett. B **374**, 181 (1996) and references therein.
[4] N.G. Deshpande and X.-G. He, Phys. Rev. Lett. **74**, 26 (1995).
[5] M. Gronau, O.F. Hernández, D. London, and J.L. Rosner, Phys. Rev. D **52**, 6374 (1995).
[6] R. Fleischer, Phys. Lett. B **365**, 399 (1996).
[7] S. Gardner, Phys. Rev. D **59**, 077502 (1999).
[8] C. Caso et al. (Particle Data Group), Eur. Phys. J C **3**, 1 (1998).
[9] The expression for $r_{f_{CP}}$ assumes that the sign of the B_B mixing parameter is positive, see Y. Grossman, B. Kayser, and Y. Nir, Phys. Lett. B **415**, 90 (1997). We ignore the width difference $\Delta, = ,_H - ,_L$.
[10] M. Gronau, D. Pirjol, and T.-M. Yan, hep-ph/9810482.
[11] D. Pirjol, hep-ph/9903447.
[12] π^0-η, η' mixing also generates significant isospin-violating corrections to ϵ'/ϵ, see A.J. Buras and J.-M. Gerard, *Phys. Lett. B* **192**, 156 (1987); J.F. Donoghue, E. Golowich, B.R. Holstein, and J. Trampetic, Phys. Lett. B **179**, 361 (1986).
[13] J.F. Donoghue, E. Golowich, and B.R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, 1992).
[14] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 539 (1985).
[15] E. S. Na and B. R. Holstein, Phys. Rev. D **56**, 4404 (1997). See also H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984).
[16] A. Ali, G. Kramer, and C.-D. Lü, Phys. Rev. D **58**, 094009 (1998).
[17] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 517 (1985).
[18] R. Fleischer, Z. Phys. C **58**, 483 (1993).
[19] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).
[20] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983). We use Eq.(10) to compute the CKM matrix elements.
[21] A. Ali, hep-ph/9801270; A. Ali and D. London, Nucl. Phys. B Proc. Suppl. **54A**, 297 (1997); F. Parodi, P. Roudeau, and A. Stocchi, hep-ph/9802289.
[22] N.G. Deshpande and J. Trampetic, Phys. Rev. D **41**, 2926 (1990); H. Simma and D. Wyler, Phys. Lett. **B272**, 395 (1991).
[23] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **29**, 637 (1985); **34**, 103 (1987).
[24] Y. Grossman and H.R. Quinn, Phys. Rev. D **58**, 017504 (1998).
[25] D. London, private communication, from an update of Ref. [21].
[26] S. Mele, Phys. Rev. D **59**, 113011 (1999).
[27] M. Ciuchini et al., Nucl. Phys. B **501**, 271 (1997); **512**, 3 (1998).