# A preliminary measurement of the gluon splitting rate into $b\bar{b}$ pairs in hadronic $Z^0$ decays

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We present a measurement of the rate of gluon splitting into bottom quarks, $g \to b\bar{b}$, in hadronic $Z^0$ decays collected by SLD from 1996 to 1998. The analysis was performed by looking for secondary bottom production in 4-jet events of any primary flavor. A topological vertex mass technique was used to tag the two jets with the smallest angle between them as $b/\bar{b}$. We obtained a rate of $g \to b\bar{b}$ per hadronic event to be $(3.07 \pm 0.71(\text{stat.}) \pm 0.66(\text{syst.})) \times 10^{-3}$ (preliminary).

## I. INTRODUCTION

The process of the splitting of a gluon into a heavy-quark pair is one of the elementary processes in QCD but is poorly known, both theoretically and experimentally.

The rate $g_{b\bar{b}}$ is defined as the fraction of hadronic events in which a gluon splits into a $b\bar{b}$ pair, $e^+e^- \to q\bar{q}g \to q\bar{q}b\bar{b}$. The value of $g_{b\bar{b}}$ is an infrared finite quantity, because the $b$-quark mass provides a natural cutoff, hence it can be safely computed in the framework of perturbative QCD [1]. However the rate is sensitive to the $\alpha_S$ parameter and to the $b$-quark mass, which results in a substantial theoretical uncertainty in the calculation of $g_{b\bar{b}}$. The limited accuracy of the $g_{b\bar{b}}$ prediction is one of the main sources of uncertainty in the measurement of the partial decay width $R_b = , (Z^0 \to b\bar{b})/, (Z^0 \to q\bar{q})$ [2,3]. In addition, about 50% of the B hadrons produced at the Tevatron are due to the gluon splitting process, and a larger fraction is expected to contribute at the LHC. A better knowledge of this process can improve theoretical predictions of heavy-flavor production at such colliders.

This measurement is difficult experimentally. The cross section of $g \to b\bar{b}$ is very small even at $Z^0$ energies, since the gluon must have sufficient mass to produce the bottom-quark pair. There are huge backgrounds from $Z^0 \to b\bar{b}$ whose magnitude is about a hundred times larger than the $Z^0 \to q\bar{q}g \to q\bar{q}b\bar{b}$ process. Moreover the B hadrons from $g \to b\bar{b}$ have relatively low energy and short flight distance and are more difficult to distinguish using standard vertexing. So far, the only three measurements of $g_{b\bar{b}}$ have been reported, by DELPHI and ALEPH [4].

Here we present a new measurement of $g_{b\bar{b}}$ based on a 400k $Z^0$-decay data sample taken in 1996-98 at the Stanford Linear Collider (SLC), with the SLC Large Detector (SLD). In this period, $Z^0$ decays were collected with an upgraded vertex detector, wider acceptance and better impact parameter resolution, thus improving considerably the $b$-tagging performance.

## II. THE SLD DETECTOR

A full description of the SLD and its performance have been described in detail elsewhere [5]. Only the details most relevant to this analysis are mentioned here.

SLD is well-suited for the measurement of $g \to b\bar{b}$ due to two unique features. The first is that the SLC, the only linear collider in the world, provides a very small and stable beam spot. The SLC interaction point was reconstructed from tracks in sets of approximately thirty sequential hadronic $Z^0$ decays with an uncertainty of only $5\mu$m transverse to the beam axis and $32\mu$m (for $b\bar{b}$ events) along the beam axis. Second is the upgraded vertex detector (VXD3) [6], a pixel-based CCD vertex detector. VXD3 consists of 3 layers with 300M pixels and each layer is only 0.36% of a radiation length thick. The measured $r\phi$ ($rz$) track impact-parameter resolution approaches $11\mu$m ($23\mu$m) for high momentum tracks, while multiple scattering contributions are $40\mu$m$/(p_\perp \sin^{3/2}\theta)$ in both projections ($z$ is the coordinate parallel to the beam axis and $p_\perp$ is the momentum in GeV/c perpendicular to the beam line). With these features, topological vertex finding gives excellent $b$-tagging efficiency and purity. In particular, the efficiency is good even at low B-meson energies, which is especially important for detecting $g \to b\bar{b}$.

## III. FLAVOR TAGGING

Topologically reconstructed secondary vertices [7] are used by many analyses at the SLD for heavy-quark tagging. To reconstruct the secondary vertices, the space points where track density functions overlap are found in 3-dimensions. Only the vertices that are significantly displaced from the primary vertex (PV) are considered to be possible B- or D-hadron decay vertices. The mass of the secondary vertex is calculated using the tracks that are associated with the vertex. Since the heavy-hadron decays are frequently accompanied by neutral particles, the reconstructed mass is corrected to account for this fact. By using kinematic information from the vertex flight path and the momentum sum of the tracks associated with the secondary vertex, we calculate the $P_T$-corrected mass $M_{P_T}$ by adding a minimum amount of missing momentum to the invariant mass, as follows:

$$M_{P_T} = \sqrt{M^2_{VTX} + P_T{}^2} + |P_T|.$$

Here $M_{VTX}$ is the invariant mass of the tracks associated with the reconstructed secondary vertex and $P_T$ is the transverse momentum of the charged tracks with respect to the B-flight direction. In this correction, vertexing resolution as well as the PV resolution are crucial. Due to the small and stable interaction point at the SLC and the excellent vertexing resolution from the SLD CCD Vertex detector, this technique has so far only been successfully applied at the SLD.

## IV. MONTE CARLO AND DATA SAMPLES

The measurement uses 400k events collected from 1996 to 1998 with the requirement that the VXD3 was fully operational.

For the purpose of estimating the efficiency and purity of the $g \to b\bar{b}$ selection procedure, we made use of a detailed Monte-Carlo simulation of the detector. The JETSET 7.4 [8] event generator was used, with parameter values tuned to hadronic $e^+e^-$ annihilation data [9], combined with a simulation of B hadron decays tuned to $\Upsilon(4S)$ data [10] and a simulation of the SLD based on GEANT 3.21 [11]. Inclusive distributions of single-particle and event-topology observables in hadronic events were found to be well described by the simulations [12]. Uncertainties in the simulation were taken into account in the systematic errors (Section VII).

Monte-Carlo events are reweighted to take into account current estimates for gluon splitting into heavy-quark pairs [4,13]. The JETSET at SLD predicts $g_{b\bar{b}} = 0.14\%$ and $g_{c\bar{c}} = 1.36\%$, and we reweighted them so that $g_{b\bar{b}} = 0.273\%$ and $g_{c\bar{c}} = 2.58\%$. A Monte-Carlo production of about 1200k $Z \to q\bar{q}$ events, 1000k $Z \to b\bar{b}$ events and 480k $Z \to c\bar{c}$ events are used in order to better evaluate the efficiencies.

Besides the signal events, hereafter called B, two categories of background events exist:

- Events which do not contain any gluon splitting into heavy flavor at all, hereafter called Q events; and

- Events in which a gluon splits to a charm quark pair, named C events.

## V. EVENT SELECTION

The two B hadrons coming from the gluon tend to be produced in a particular topological configuration, which allows one to discriminate the signal from background. We select $g \to b\bar{b}$ events as follows:

- Require 4 jets in the events;

- Require $b$ tags in two jets selected in a particular configuration; and

- Apply additional topological selections to improve the signal/background ratio.

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Jets are formed with energy-flow particles, using the Durham jet-finding algorithm [14] with $y_{cut} = 0.008$, chosen to minimize the statistical error. The 4-jet fractions for the B, C and Q events predicted by the simulation are about 32%, 18% and 5.3%, respectively. The overall 4-jet rate in the data is $(5.976 \pm 0.044)\%$, where the error is statistical only. In the Monte-Carlo simulation the rate is $(5.678 \pm 0.002 \pm 0.068)\%$ where the first error is statistical and the second is due to the uncertainty in the simulation of heavy-quark-hadron physics. The two jets forming the smallest angle in the event are considered as candidates for originating from the gluon splitting process $g \to b\bar{b}$. The selected jets are labeled as jet 1 and jet 2, where jet 1 is more energetic than jet 2. The other two jets in the event are labeled as jets 3 and 4, where jet 3 is more energetic than jet 4.

Jets containing B-hadron decay products are then searched for by making use of the information coming from the vertex detector, using the topological vertex method. We require jet 1 and jet 2 to each have a secondary vertex. No tag is applied to jet 3 and jet 4. After topological vertexing, about 300 events are selected. The selection efficiency for $g \to b\bar{b}$ is expected from Monte Carlo to be 6.6% while the signal/background ratio is 1/5. 67% of the background comes from $Z \to b\bar{b}$ events, 21% from $g \to c\bar{c}$ events and remaining 12% from $Z \to q\bar{q}$ ($q \neq b$) events.



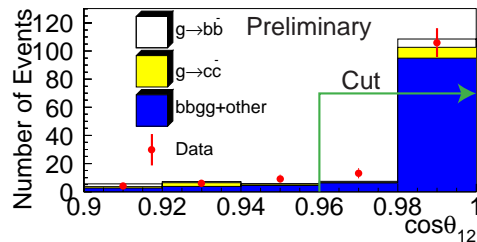FIG. 1. Angular distribution between vertex axes in jet 1 and jet 2 ($0.9 < \cos\theta_{12}$). Points indicate data, open box signal, hatched boxes are backgrounds.

In order to improve the signal/background ratio, we use topological information. Firstly, many $b\bar{b}$ background have one $b$-jet splitting into 2 jets so that the two found vertices are from different decay products from the same B decay. The two vertex axes tend to be collinear. Figure 1 shows the angular distribution between vertex axes in jet 1 and jet 2. Half of the $b\bar{b}$ background peaks at $\cos\theta_{12} \sim 1$. In order to remove $b\bar{b}$ events, we require $-0.2 < \cos\theta_{12} < 0.96$.
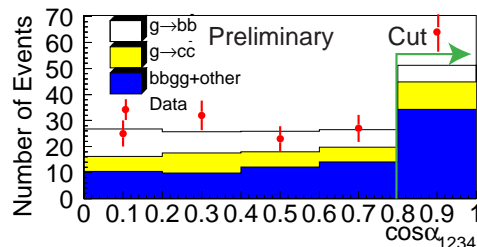


FIG. 2. The distribution of the cosine of angle between the plane $\Pi_{12}$ formed by jets 1 and 2 and the plane $\Pi_{34}$ formed by jets 3 and 4, for data (points) and Monte Carlo (histogram). We reject $|\cos\alpha_{1234}| > 0.8$

Secondly, the variable $|\cos\alpha_{1234}|$, where $\alpha_{1234}$ is the angle between the plane $\Pi_{12}$ formed by jets 1 and 2 and the plane $\Pi_{34}$ by jets 3 and 4, is used to suppress the $b\bar{b}$ background. Figure 2 shows the distribution of $|\cos\alpha_{1234}|$. This variable is similar to the Bengtsson-Zerwas angle [15], and is useful to separate $g \to b\bar{b}$ events because the radiated virtual gluon in the process $Z^0 \to q\bar{q}g$ is polarized in the plane of the three-parton event, and this is reflected in its subsequent splitting, by strongly favoring $g \to q\bar{q}$ emission out of this plane. Events with $|\cos\alpha_{1234}| > 0.8$ are rejected.

Thirdly, the $b$ jets coming from a gluon tend to have lower energy than the other two jets in the event. We require the jet-energy sum of jet 1 and jet 2 to be smaller than 36 GeV.
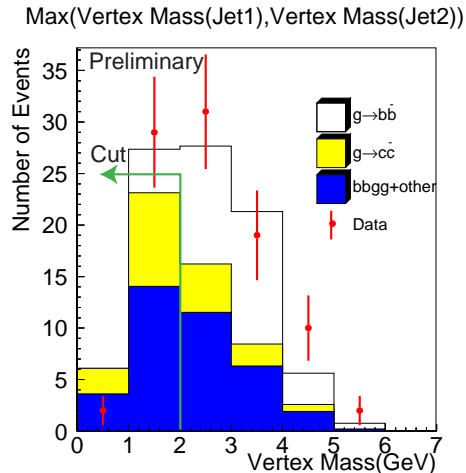
FIG. 3. Maximum $P_T$-corrected mass distribution between jet 1 and jet 2 after jet-energy-sum cut. Points indicate data, open box signal, hatched boxes are backgrounds.

Finally, $c$ jets have lower $P_T$-corrected mass than $b$ jets. Figure 3 shows the greater of the $P_T$-corrected mass determined for jet 1 and jet 2 after the above cuts. Many $g \to c\bar{c}$ events are in below 2.0 GeV. Hence we require maximum $P_T$-corrected mass to be greater than 2.0 GeV to remove $g \to c\bar{c}$ events.

## VI. RESULT

After requiring all the above mentioned cuts, 62 events are selected in the data. Background events are estimated to be 27.6 using Monte Carlo, where 63% of the background comes from $Z \to b\bar{b}$ events, 27% from $g \to c\bar{c}$ events and the remaining 10% from $Z \to q\bar{q}$ ($q \neq b$) events. Table I shows the tagging efficiencies for the three categories of events, where the errors are statistical only. From these efficiencies and the fraction of events selected in the data $f_d = (2.14 \pm 0.27) \times 10^{-4}$, the value of $g_{b\bar{b}}$ can be extracted as:

$$g_{b\bar{b}} = \frac{f_d - (1 - g_{c\bar{c}})\epsilon_Q - g_{c\bar{c}}\epsilon_C}{\epsilon_B - \epsilon_Q}. \tag{1}$$

The measured value of the gluon splitting rate into $b\bar{b}$ pairs is

$$g_{b\bar{b}} = (3.07 \pm 0.71) \times 10^{-3}, \tag{2}$$

where the error is statistical only.

TABLE I. Efficiencies after all cuts for the three categories. Errors are statistical only.

| Source | Efficiency (%) |
|---|---|
| B | $3.86 \pm 0.52$ |
| C | $0.10 \pm 0.02$ |
| Q | $0.73 \pm 0.05$ |

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# VII. SYSTEMATIC ERROR

The efficiencies for the three event categories are evaluated by Monte-Carlo simulation. The limitations of the simulation in estimating these efficiencies lead to an uncertainty on the result. The error due to the limited Monte-Carlo statistics in the efficiency evaluation is $\Delta g_{b\bar{b}} = \pm 0.44 \times 10^{-3}$. This uncertainty comes mainly from the efficiency to tag Q events.

A large fraction of events remaining after the selection cuts contain $b$ and $c$ hadrons. The uncertainty in the knowledge of the physical processes in the simulation of heavy-flavor production and decays constitutes a source of systematic error. All the physical simulation parameters are varied within their allowed experimental ranges. In particular, the $b$ and $c$ hadron lifetimes as well as production rates are varied, following the latest recommendations of the LEP Heavy Flavour Working Group [16]. The uncertainties are summarized in Table II.

The simulation of the signal events is based on the JETSET parton shower Monte Carlo, which is in good agreement with the theoretical predictions [1]. In order to estimate the uncertainty on this assumption, we have produced 10,000 $g \rightarrow b\bar{b}$ events using GRC4F [17] at the generator level. The signal tagging efficiency mainly depends on the energy of the gluon splitting into $b\bar{b}$. This efficiency function, computed with JETSET, is reweighted by the ratio of GRC4F to JETSET initial distributions to obtain the average efficiency. A systematic error of $\pm 0.09 \times 10^{-3}$ is estimated from the difference in $\epsilon_B$ from the two Monte-Carlo models.

The dependence of the B efficiency on the $b$-quark mass has also been investigated at the generator level. Events are generated using the GRC4F Monte Carlo, which is based on a matrix element calculation including $b$-quark masses. The variation of the B efficiency is computed as done for JETSET, using the GRC4F spectrum for $b$-quark masses from 4.7 and 5.3 GeV/c$^2$. The uncertainty is estimated to be $0.06 \times 10^{-3}$.

The uncertainty in the ratio of the $g \rightarrow c\bar{c}$ background events, $\Delta g_{c\bar{c}} = \pm 0.40\%$, gives the error $\Delta g_{b\bar{b}} = \pm 0.09 \times 10^{-3}$.

There is about 5% discrepancy of 4-jet rate between data and Monte Carlo in our $y_{cut}$. The uncertainty due to the discrepancy is estimated by increasing background events in the Monte Carlo to be $\Delta g_{b\bar{b}} = \pm 0.14 \times 10^{-3}$.

Charged Monte-Carlo tracks used by the topological vertex tag are smeared and tossed to better reproduce distribution of data. Uncertainties in the efficiencies due to this smearing and tossing are assessed by evaluating the Monte-Carlo efficiencies without the smearing and tossing algorithm. The difference in the $g_{b\bar{b}}$ result is taken as systematic error. The errors on $g_{b\bar{b}}$ due to the tracking resolution and efficiency are then estimated as $\Delta g_{b\bar{b}} = \pm 0.26 \times 10^{-3}$ and $= \pm 0.29 \times 10^{-3}$, respectively.

Table II summarizes the different sources of systematic error on $g_{b\bar{b}}$, and the total systematic error is estimated to be $0.66 \times 10^{-3}$.

TABLE II. Systematic uncertainties on $g_{b\bar{b}}$.

| Source | $\Delta g_{b\bar{b}}$ ($10^{-3}$) |
|---|---|
| Monte Carlo statistics | $\pm 0.44$ |
| $b$ hadron lifetimes | $\pm 0.01$ |
| $b$ hadron production | $\pm 0.07$ |
| $b$ hadron fragmentation | $\pm 0.12$ |
| $b$ hadron charged multiplicities | $\pm 0.11$ |
| $c$ hadron lifetimes | $\pm 0.01$ |
| $c$ hadron production | $\pm 0.03$ |
| $c$ hadron charged multiplicities | $\pm 0.08$ |
| Energy distribution of $g \rightarrow b\bar{b}$ | $\pm 0.08$ |
| $b$ quark mass | $\pm 0.06$ |
| $g_{c\bar{c}}$ | $\pm 0.09$ |
| 4-jet rate discrepancy | $\pm 0.14$ |
| IP resolution | $\pm 0.09$ |
| Track resolution | $\pm 0.26$ |
| Tracking efficiency | $\pm 0.29$ |
| Total (Preliminary) | $\pm 0.66$ |

## VIII. SUMMARY

A measurement of the gluon splitting rate to a $b\bar{b}$ pair in hadronic $Z^0$ decays collected by SLD has been presented. Excellent SLC and VXD3 performance provides advantages not only for $b$-tag efficiency but also for topological selections. The result is

$$g_{b\bar{b}} = (3.07 \pm 0.71\,(\text{stat.}) \pm 0.66(\text{syst.})) \times 10^{-3}(\text{preliminary}).$$

where the first error is statistical and the second includes all systematic effects.

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[1] D.J. Miller and M.H. Seymour, Phys. Lett. **B435**, 213 (1998).

[2] K. Abe *et al.* [SLD Collaboration], Phys. Rev. Lett. **80**, 660 (1998);
    K. Abe *et al.* [SLD Collaboration], Phys. Rev. **D53**, 1023 (1996).

[3] R. Barate *et al.* [ALEPH Collaboration], Phys. Lett. **B401**, 150 (1997);
    R. Barate *et al.* [ALEPH Collaboration], Phys. Lett. **B401**, 163 (1997);
    P. Abreu *et al.* [DELPHI Collaboration], Z. Phys. **C70**, 531 (1996);
    P. Abreu *et al.* [DELPHI Collaboration], Z. Phys. **C66**, 323 (1995);
    O. Adriani *et al.* [L3 Collaboration], Phys. Lett. **B307**, 237 (1993);
    G. Abbiendi *et al.* [OPAL Collaboration], Eur. Phys. J. **C8**, 217 (1999).

[4] R. Barate *et al.* [ALEPH Collaboration], Phys. Lett. **B434**, 437 (1998);
    P. Abreu *et al.* [DELPHI Collaboration], Phys. Lett. **B405**, 202 (1997);
    P. Abreu *et al.* [DELPHI Collaboration], ICHEP'98 contributed paper #776

[5] SLD Design Report, SLAC Report 273 (1984).

[6] K. Abe *et al.*, Nucl. Instrum. Meth. **A400**, 287 (1997).

[7] D. J. Jackson, Nucl. Inst. and Meth. **A388**, 247 (1997).

[8] T. Sjöstrand, Comput. Phys. Commun. **82**, 74 (1994).

[9] P. N. Burrows, Z. Phys. **C41** 375 (1988).
    OPAL Collab., M.Z. Akrawy *et al.*, Z. Phys. **C47** 505 (1990).

[10] SLD Collab., K. Abe *et al.*, SLAC-PUB-7117; to appear in Phys. Rev. Lett.

[11] R. Brun *et al.*, Report No. CERN-DD/EE/84-1 (1989).

[12] SLD Collaboration, K. Abe *et al.*, Phys. Rev. **D51,** 962 (1995).

[13] O. Adriani *et al.* [L3 Collaboration], ICHEP'98 contributed paper #539;
    R. Akers *et al.* [OPAL Collaboration], Phys. Lett. **B353**, 595 (1995);
    R. Akers *et al.* [OPAL Collaboration], Z. Phys. **C67**, 27 (1995).

[14] S Catani *et al.*, Phys. Lett. **B269,** 432 (1991).

[15] M. Bengtsson and P.Zerwas, Phys. Lett. **B208,** 306 (1988).

[16] The LEP Heavy Flavour Working Group, LEPHF/98-01.

[17] J. Fujimoto *et al.*, Comput. Phys. Commun. **100**, 128 (1997).