

Hadronic Three Jet Production at Next-to-Leading Order

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We report preliminary results for a next-to-leading order event generator for hadronic three jet production. We demonstrate the stability of the calculation and present preliminary results for the jet transverse energy spectra. This is the first calculation of three jet production at this order to include all parton sub-processes.

I. INTRODUCTION

In this talk I will discuss recent work in constructing a next-to-leading order event generator for hadronic three jet production. This is the first calculation of three jet production at this order to include all parton sub-processes. Previous studies [1,2] have only included the contributions of pure gluon scattering contribution. As my preliminary results show, the generator is now working properly and is ready to perform phenomenological studies.

II. MOTIVATION

When interpreting experimental data, one would like to have some understanding of the uncertainty associated with theoretical expectations. In QED and the weak interactions this is not a big problem because the couplings are sufficiently weak that higher order corrections are generally quite small. In QCD, however, the coupling is quite strong (α_s is still of order 1/8 at the scale of the Z boson mass) and it is difficult to obtain a reliable estimate of the theoretical uncertainty. Typically, one characterizes theoretical uncertainty by the dependence on the renormalization scale μ . Since we don't actually know how to choose μ or even a range of μ , the uncertainty associated with scale dependence is somewhat arbitrary. There seems no way around this other than to calculate to higher order where the scale dependence is expected to be smaller.

However, one often obtains other improvements besides reduced scale dependence by going to higher order. Sometimes one finds the next-to-leading order (NLO) correction to be very large. A notorious example is Higgs production at hadron colliders where the NLO corrections to the leading order gluon fusion process are of the order of 100%. Such large corrections often come from opening up new channels that are forbidden at leading order (LO), but one must still be concerned about the question of perturbative convergence. Regardless of the nominal scale dependence, one simply does not know how to trust a calculation when the perturbative corrections are large.

Even if the overall NLO correction is relatively small, there may be regions of phase space, typically near the boundaries of the allowed region for the LO process, where NLO corrections are large. In these regions, NLO calculations are effectively of leading order and suffer from the large scale dependence associated with leading order. It is only in those regions of phase space where the NLO corrections are well behaved (as determined by the ratio of the NLO to LO terms) that one has confidence in the reliability of the calculation and can begin to believe the uncertainty estimated from scale dependence and it is only when one has a reliable estimate of the theoretical uncertainty that comparisons to experiment are meaningful.

A next-to-leading order three jet calculation will have many phenomenological applications. One of the most important will be to perform a purely hadronic extraction of α_s via the ratio of two-jet to three-jet production. Because these processes have the same production mechanisms, such an extraction should be relatively free of parton distribution uncertainties. Since hadron machines produce events at all accessible energy scales, it should be possible to measure the running of α_s and thereby the QCD β function which depends upon the strongly interacting matter content accessible at each scale.

This calculation will also be useful for studying jet algorithms. One would like to have a flexible jet algorithm that makes it easy to compare experimental results with theoretical calculations. The presence of as many as four final state partons permits more complicated clustering conditions and tests the details of the algorithms. In our pure gluon study [2], we found that the iterative cone algorithms commonly used at hadron colliders have an intrinsic infrared sensitivity that precludes their direct implementation in fixed order calculations. With Run II at the Tevatron fast approaching, it would be desirable to settle on an algorithm suitable for both theory and experiment.

Other applications include the study of energy flow within jets and background studies for new phenomena searches. Finally, this entire calculation is but a part of an eventual next-to-next-to-leading order (NNLO) calculation of two-jet production. That calculation is still a long way off. Not only are the two-loop virtual corrections unknown, but even the higher-order contributions to real emission are unknown. Still, the NNLO calculation will eventually be needed to compare to the high statistics data that will be collected at the Tevatron and the LHC.

III. METHODS

The NLO three jet calculation consists of two parts: two to three parton processes at one-loop (the virtual terms) and two to four parton processes (the real emission terms) at tree-level. Both of these contributions are infrared singular; only the sum of the two is infrared finite and meaningful. The virtual contributions are infrared singular because of loop momenta going on-shell. The virtual singularities take the form of single and double poles in the dimensional regulator ϵ multiplying the Born amplitude. The real emission contributions are singular when two partons become collinear or when a gluon becomes very soft. The Kinoshita-Lee-Nauenberg (KLN) theorem [3] guarantees that the infrared singularities cancel for sufficiently inclusive processes when the real and virtual contributions are combined.

The parton sub-processes involved are $gg \rightarrow ggg$ [4], $\bar{q}q \rightarrow ggg$ [5], $\bar{q}q \rightarrow \bar{Q}Qg$ [6], and processes related to these by crossing symmetry, all computed to one-loop, and $gg \rightarrow gggg$, $\bar{q}q \rightarrow gggg$, $\bar{q}q \rightarrow \bar{Q}Qgg$, and $\bar{q}q \rightarrow \bar{Q}Q\bar{Q}'Q'$ and the crossed processes computed at tree-level. Quark-antiquark pairs $Q\bar{Q}$ may or may not have the same flavor as the $q\bar{q}$ and $Q'\bar{Q}'$ pairs. Previous NLO three jet calculations have worked in the approximation of pure gluon scattering, using only the $gg \rightarrow ggg$ and $gg \rightarrow gggg$ processes. This is the first calculation to include all parton sub-processes.

In order to implement the kinematic cuts necessary to compare a calculation to experimental data one must compute the cross section numerically. Thus, it is not sufficient to know that the singularities drop out in the end, we must find a way of canceling them before we start the calculation. The crucial issue in obtaining and implementing the cancelation is resolution. The real emission process is infrared singular in precisely those regions where the individual partons cannot all be resolved (even in principle, ignoring the complication of hadronization, showering, *etc.*) because of collinear overlap or by becoming too soft to detect. If we impose some resolution criterion, we can split the real emission calculation into two parts, the “hard emission” part in which all of the partons are well resolved and the “infrared” part in which one or more partons are unresolved.

The hard emission part is computed in the normal way by means of Monte Carlo integration. The infrared part is treated differently, making use of the fact that matrix elements have well defined factorization properties in both soft and collinear infrared limits. In terms of color-ordered helicity amplitudes,

$$\mathcal{M}_n(\dots, 1^{\lambda_1}, 2^{\lambda_2}, \dots) \xrightarrow{1\parallel 2} \text{Split}_{-\lambda_c}(1^{\lambda_1}, 2^{\lambda_2})\mathcal{M}_{n-1}(\dots, c^{\lambda_c}, \dots) \quad (1)$$

$$\mathcal{M}_n(\dots, 1^{\lambda_1}, s^{\lambda_s}, 2^{\lambda_2}, \dots) \xrightarrow{k_s \rightarrow 0} \text{Soft}(1, s^{\lambda_s}, 2)\mathcal{M}_{n-1}(\dots, 1^{\lambda_1}, 2^{\lambda_2}, \dots),$$

where Split and Soft are universal functions depending only on the momenta, helicities and particle types involved. The Split functions are in a sense the square roots of the Altarelli-Parisi splitting functions. In computing the infrared part, we replace the full two-to-four parton matrix elements with their infrared factorized limits. We then integrate out the unresolved parton by integrating (in dimensional regularization) the Split and Soft functions over the unresolved region of phase space, resulting in single and double poles multiplying two-to-three parton Born matrix elements.

These terms, as the KLN theorem says they must, have exactly the right pole structure to cancel the infrared poles of the virtual contribution. By analytically combining unresolved real emission with the virtual terms, we obtain a finite contribution that can be integrated numerically.

Several different methods [7–12,2] of implementing this infrared cancelation have successfully employed in various NLO calculations. The method we use is the “subtraction improved” phase space slicing method [2]. The phase space slicing method [9,10] uses a resolution criterion s_{min} , which is a cut on the two parton invariant masses,

$$s_{ij} = 2E_i E_j (1 - \cos \theta_{ij}). \quad (2)$$

If partons i and j have $s_{ij} > s_{min}$ they are said to be resolved from one another. (Which is not to say that a jet clustering algorithm will not put them into the same jet.) If $s_{ij} < s_{min}$ partons i and j are said to be unresolvable. One advantage of the s_{min} criterion is that it simultaneously regulates both soft ($E_i \rightarrow 0$ or $E_j \rightarrow 0$) and collinear ($\cos \theta_{ij} \rightarrow 1$) emission. In the rearrangement of terms, the infrared region of phase space is where any two parton invariant mass is less than s_{min} . These regions are sliced out of the full two-to-four body phase space, partially integrated and then added to the two-to-three body integral.

Because the infrared integral is bounded by s_{min} , the integrations over Split and Soft terms depend explicitly on s_{min} . In fact, in the cancelation of the virtual singularities, the $1/\epsilon$ terms are replaced by $\ln s_{min}$ terms and the $1/\epsilon^2$ terms by $\ln^2 s_{min}$ terms. The hard real emission term is also s_{min} dependent because the boundary of the sliced out region depends on s_{min} . Because s_{min} is an arbitrary parameter the sum of the virtual and real emission terms must be s_{min} independent. Thus, we have rearranged the calculation, trading a cancelation of infrared poles in ϵ for a cancelation of logarithms of s_{min} . This provides an important cross check on our calculation. If we can demonstrate that our calculated cross section is s_{min} independent we can be confident that we have correctly implemented the infrared cancelation.

While the NLO cross section is formally independent of s_{min} , there are several practical considerations to choosing the value properly. As s_{min} becomes smaller, the infrared approximations of the matrix elements becomes more accurate. However, the overriding concern is the numerical convergence of the calculation. Two terms, each diverging like $\ln^2 s_{min}$ must be added with the logs canceling. As s_{min} is made small, the logarithm becomes large and the individual terms, real and virtual, become larger in magnitude. The sum however, the NLO cross section, is unchanged. Thus, as s_{min} becomes small, it becomes harder to engineer the cancelation to the precision to which one would like to compute the cross section. Based on this consideration, we would like to make s_{min} as large as possible. There is an absolute upper limit imposed by the constraints of jet finding. We cannot make s_{min} so large that it begins to interfere with jet clustering, say, by declaring unresolvable a pair of partons that a sensible jet clustering algorithm would say are separated from one another.

Another problem with large values of s_{min} , alluded to before and which actually sets in at a lower scale than jet clustering interference, is that as s_{min} is made larger the infrared approximations used in the slicing region become less precise. The “subtraction improved” part of our method involves the handling of the slicing region. As originally implemented, the infrared region was completely sliced out of the two-to-four integration and the full two-to-four matrix element was replaced by its soft or collinear limit. A better approximation is to leave the infrared regions in the “hard” phase space integral, but to compute only the difference between the true and approximate matrix elements in those regions. In our gluonic three jet production study [2] we found that the subtraction improvement allowed us to use substantially larger values of s_{min} than would have been possible with just phase space slicing.

IV. PRELIMINARY RESULTS

The results presented below were computed for the following kinematic configurations: the $\bar{p}p$ center of mass energy is 1800 GeV; the leading jet is required to have at least 100 GeV in transverse energy, E_T , and there must be two additional jets with at least 50 GeV of transverse energy; all jets must lie in the pseudorapidity range $-4.0 < \eta_J < 4.0$.

The CTEQ3M parton distribution functions [13] and the EKS [14] jet clustering algorithm (modified for three jet configurations as in reference [2]) were used.

Figure 1 shows the computed next-to-leading order three jet cross section as a function of the resolution parameter s_{min} . Also shown is the leading order calculation.

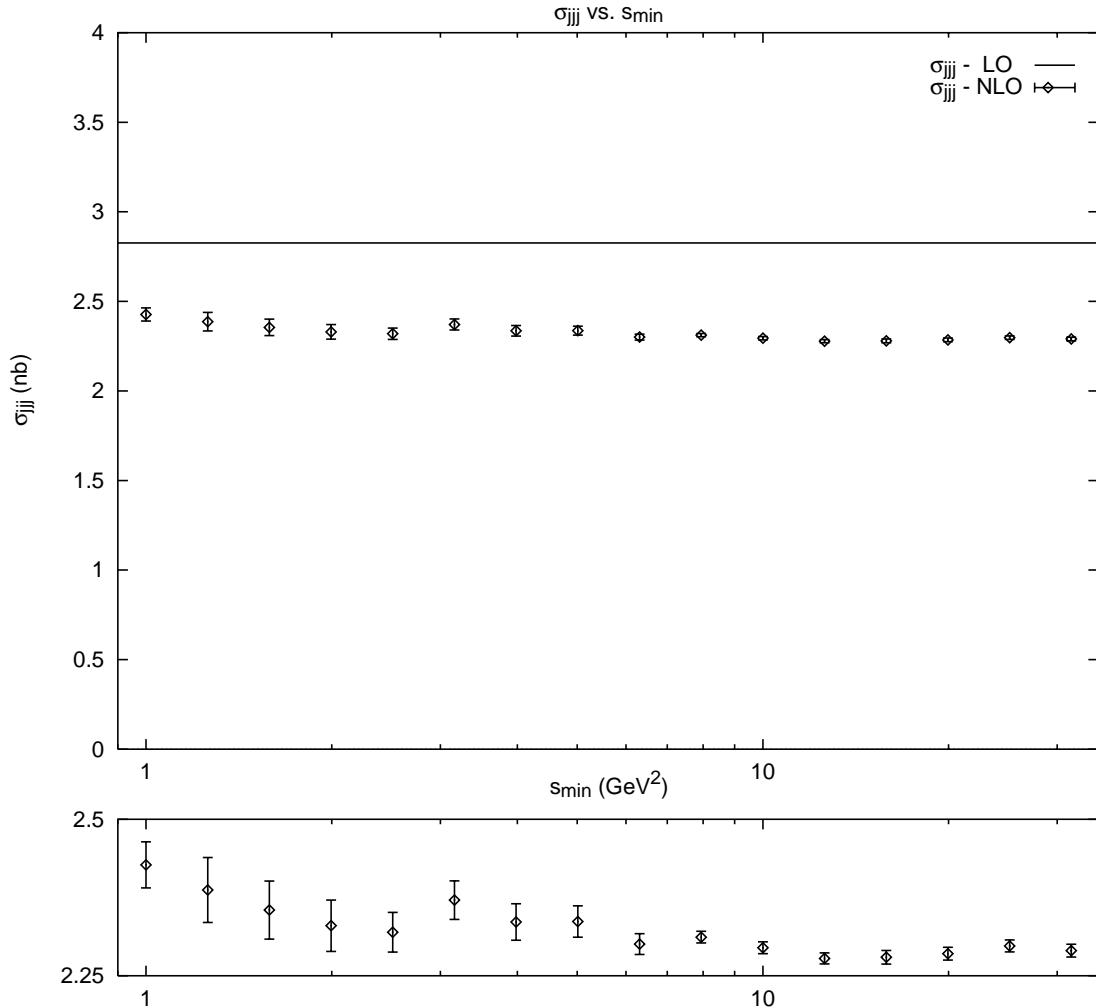


FIG. 1. Top: Next-to-leading order three jet cross section vs. s_{min} . The leading order cross section is shown as a solid line. Bottom: Expanded view showing relative statistical uncertainties.

We see that the NLO result is stable over a wide range of values of s_{min} . This stability indicates that we are correctly implementing the infrared cancelation. Further calculations at larger values of s_{min} are needed to actually determine the limit of the region of stability. In the lower plot, we see the statistical uncertainty on each point. As s_{min} becomes small, it becomes increasingly difficult to calculate σ_{jjj} to the desired precision. For instance, at $s_{min} = 10 \text{ GeV}^2$, the real and virtual components are 16.763 ± 0.008 and $-14.468 \pm 0.005 (nb)$ respectively, while at $s_{min} = 1 \text{ GeV}^2$, they are 29.739 ± 0.035 and -27.312 ± 0.010 . To obtain the same absolute uncertainty on the sum of these numbers, the relative uncertainty on each of the components at $s_{min} = 1 \text{ GeV}^2$ must be one half that required at $s_{min} = 10 \text{ GeV}^2$. Since the statistical uncertainty scales like the square root of number of points evaluated, it takes roughly four times as long to obtain a precise calculation at $s_{min} = 1 \text{ GeV}^2$ as it does at $s_{min} = 10 \text{ GeV}^2$.

We also see that the size of the NLO correction is of order 15%. This gives us confidence in the perturbative stability of the calculation. Together, these two observations indicate that we are performing a reliable calculation of the three jet cross section. Further tests using a variety of modern parton distribution functions, values of α_s , renormalization scales, *etc.* are needed to obtain a clearer picture of the theoretical uncertainty associated with the calculation.

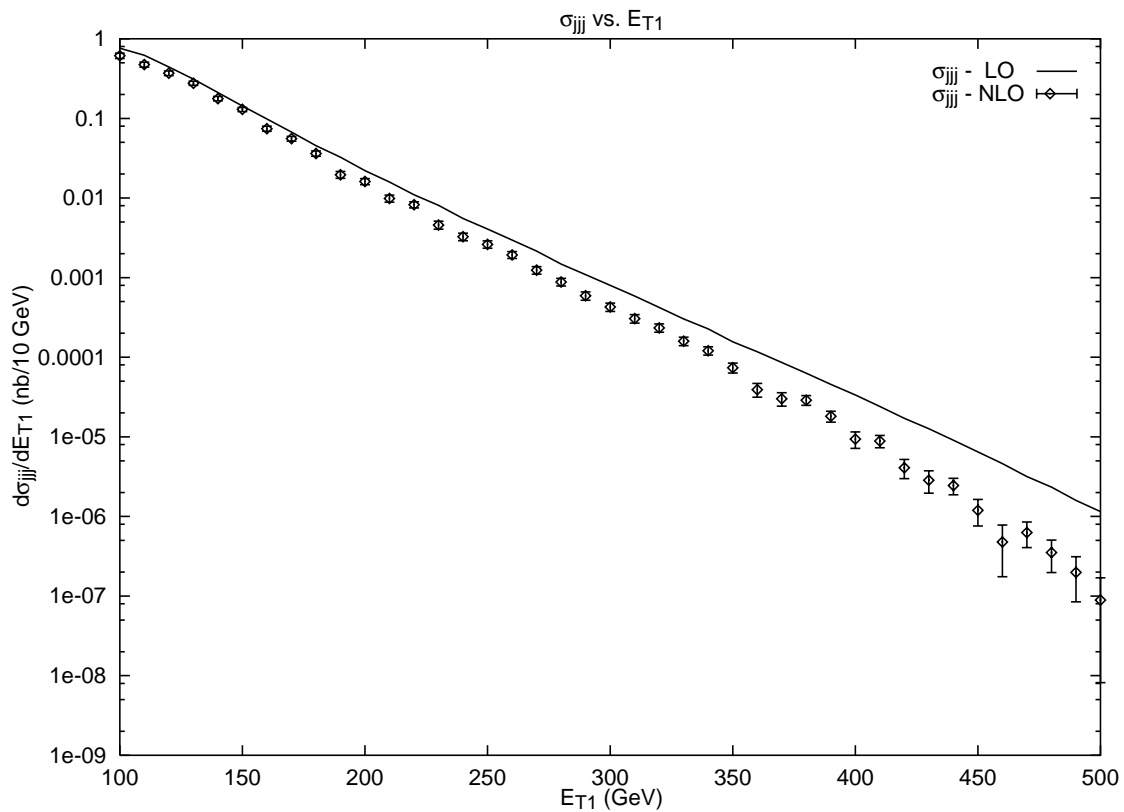


FIG. 2. Transverse energy spectrum of the leading jet. The leading order result is shown as a solid line.

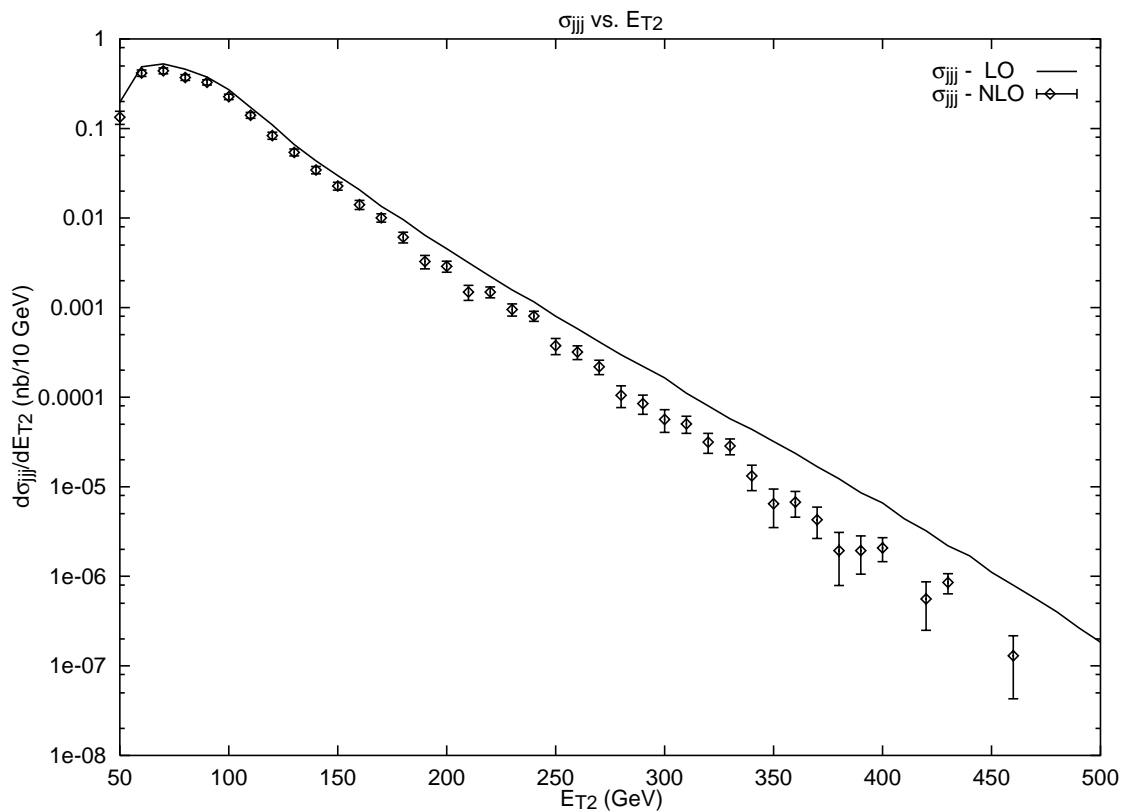


FIG. 3. Transverse energy spectrum of the second-leading jet. The leading order result is shown as a solid line.

Figure 2 shows the transverse energy spectrum of the leading jet in E_T . There is no indication of any large correction appearing in the jet spectrum. The dominant feature of the NLO spectrum is that it is somewhat softer than the LO spectrum. That is, NLO predicts that the spectrum falls more quickly with growing transverse energy than LO. This is explained in part by the fact that NLO opens up the available phase space by allowing a fourth jet in the final state. This same softening trend is also observed in the transverse energy spectrum of the second leading jet, shown in figure 3. Both jet spectra were computed at $s_{min} = 7.9 \text{ GeV}^2$.

V. CONCLUSIONS

We have successfully built a next-to-leading order event generator for inclusive three jet production at hadron colliders. This is the first NLO calculation of this process to include all parton sub-processes. Our results indicate that we are correctly canceling the infrared singularities and therefore obtaining reliable results. With this calculation we will be able to study many interesting phenomena within QCD.

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