A non-local OPE for hard QCD processes near the elastic limit

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A leading twist expansion in terms of bilocal operators is proposed for the structure functions of deeply inelastic scattering near the elastic limit $x \to 1$, which is also applicable to a range of other hard quasi-elastic processes. Operators of increasing dimensions contribute to logarithmically enhanced terms, which are suppressed by corresponding powers of $1-x$. For the longitudinal structure function in moment (N) space all the logarithmic contributions of order $\ln^k N/N$ are shown to be resummable in terms of the anomalous dimension of the leading operator in the expansion.

I. INTRODUCTION

The treatment of hard processes in QCD perturbation theory involves the factorization of the short from the long distance dynamical regimes, characterized by mass scales Q and $\Lambda_{\rm QCD}$ respectively. Such processes become quasi-elastic in those regions of phase space where a final state invariant mass M becomes much smaller than Q. For the perturbative treatment of these three-scale processes the condition $M^2 \gg \Lambda_{\rm QCD}^2$ is also necessary. Typical quasi-elastic processes are deeply inelastic scattering (DIS) at large Bjorken x, Drell-Yan production near the partonic threshold and thrust in e^+-e^- annihilation near the two-jet limit. We argue that existing methods for studying quasielastic processes can be extended in a systematic way, via a non-local operator expansion to be introduced below. The DIS longitudinal structure function will be considered for definiteness.

The longitudinal structure function (F_L), like any DIS observable at large Q, may be treated by the Wilson OPE, organized through the light cone expansion or, equivalently, factorization formulas. Given regulator (ϵ) and factorization scheme ($\overline{\rm MS}$), the leading twist factorization at scale μ^2 can be written as

$$F_L(x,Q^2,\epsilon) = \bar{C}_L(x,Q^2/\mu^2,\alpha_s(\mu^2)) \otimes f(x,\alpha_s(\mu^2),\epsilon) + \mathcal{O}(\Lambda_{\rm QCD}^2/Q^2)\,, \tag{1}$$

where we define the convolution in longitudinal momentum as

$$h(x)\otimes f(x) = \int_0^1 dx_1 \int_0^1 dx_2\; \delta(x-x_1x_2)\; h(x_1)\; f(x_2)\,, \tag{2}$$

and the perturbative expansion of any function R as $R = \sum_{n=0}^{\infty}(\alpha_s/\pi)^n R^{(n)}$. The coefficient functions can be computed in perturbation theory and the results to the second order in the coupling are [1–3]

$$\begin{aligned}
&\bar{C}_L^{(0)}(x,Q^2/\mu^2) = 0\,, \qquad\qquad \bar{C}_L^{(1)}(x,Q^2/\mu^2) = C_F x,\\
&\bar{C}_L^{(2)}(x,Q^2/\mu^2) = \left[\frac{1}{2}\left(P_{qq}^{(1)}\otimes \bar{C}_L^{(1)}\right)(x) - \frac{1}{4}\beta_1\,\bar{C}_L^{(1)}(x)\right]\ln\frac{Q^2}{\mu^2} + \bar{C}_L^{(2)}(x,1)\,,
\end{aligned} \tag{3}$$

with P_{qq} the quark splitting function and β_1 the one-loop coefficient of the beta function. The coefficient function at $Q^2=\mu^2$ is

$$\bar{C}_L^{(2)}(x,1) = \frac{1}{2}C_F^2\ln^2(1-x) + \left[\frac{9}{4}-2\zeta(2)\right]C_F^2\ln(1-x) + \left[(\zeta(2)-1)C_FC_A - \frac{1}{4}\beta_1 C_F\right]\ln(1-x) + {\rm reg}\,. \tag{4}$$

Only terms non-analytic at $x=1$ are displayed in the above equation. Our objective is to show that these terms can be captured by expectations of appropriately defined operators, and that they can be resummed to all orders in perturbation theory. It is instructive at this point to compare the above coefficient function with the corresponding one for F_2 to $\mathcal{O}(\alpha_s)$,

$$\bar{C}_2^{(1)}(x, Q^2/\mu^2) = \frac{1}{2} P_{qq}^{(1)} \ln \frac{Q^2}{\mu^2} + 4C_F \left(\frac{\ln(1-x)}{1-x}\right)_+ - 3\frac{1}{(1-x)_+} + 4C_F \ln(1-x) + \text{reg.} \tag{5}$$

We note the absence of plus distributions in the case of F_L. The longitudinal structure function has only logarithmic divergences to all orders in perturbation theory. Upon taking the Mellin transform

$$\tilde{F}(N, Q^2) = \int_0^1 dx\, x^{N-1}\, F(x, Q^2)\,, \tag{6}$$

the leading logarithms of F_L are of order $\ln^k N/N$ in moment space. Near the elastic limit ($N \to \infty$) they are power suppressed relative to the order $\ln^k N$ terms coming from the plus distributions of F_2. For this reason, only recently has the Sudakov factorization formalism been extended to capture purely logarithmic threshold corrections [4].

II. THE NON-LOCAL OPE IN THE ELASTIC LIMIT

The Sudakov factorization formalism of perturbative QCD applies to hard processes in the quasielastic regime $Q^2 \gg M^2 \gg \Lambda^2_{\rm QCD}$ and resums terms of order $\ln^k N$, where N is the moment with respect to the variable $(Q^2 - M^2)/Q^2$. We shall refer to this formalism as the leading-jet approximation, and we briefly summarize it here before extending it to the case of F_L. (For a recent review and references see Ref. [5]).

Every DIS observable is obtained as a particular projection of the hadronic tensor

$$W_{\mu\nu}(p, q) = \frac{1}{4\pi} \int d^4y\, \mathrm{e}^{-iq\cdot y}\, \langle p| j^\dagger_\mu(0)\, j_\nu(y)|p\rangle\,. \tag{7}$$

Structure functions are obtained as $F_L = P_L^{\mu\nu} W_{\mu\nu}$, $F_2 = P_2^{\mu\nu} W_{\mu\nu}$ with the projectors

$$P_L^{\mu\nu} = \frac{8x^2}{Q^2} p^\mu p^\nu\,, \qquad P_2^{\mu\nu} = -\eta^{\mu\nu} + \frac{3}{2} P_L^{\mu\nu}\,. \tag{8}$$

The leading-jet expansion of F_2 leads to the following factorization formula

$$F_2(x, Q^2) = |H_2(Q^2)|^2 \int_x^1 dx' \int_0^{x'-x} dw\; J\left((x'-x-w)Q^2\right)\; V(w)\; \phi(x')\; \Big(1 + \mathcal{O}(1-x)\Big)\,. \tag{9}$$

All three factors in the longitudinal momentum convolution integral are defined as expectation values of non-local operators. Explicitly, the soft function is

$$V(w) = \int dy^- \mathrm{e}^{-iwy^- p\cdot\bar{v}}\, \langle 0| \Phi_v^\dagger(0, -\infty)\; \Phi_{\bar{v}}(0, y^-\bar{v})\; \Phi_v(y^-\bar{v}, -\infty)|0\rangle\,, \tag{10}$$

the initial state jet function is

$$\phi(x') = \int dy^-\; e^{-ix'y^- p\cdot\bar{v}} \langle p|\psi(0)\; \not{\bar{v}}\; \Phi_{\bar{v}}(0, y^-)\, \bar{\psi}(y)|p\rangle \otimes V^{-1}\,, \tag{11}$$

and the final state jet function is

$$J\left((1-z)Q^2\right) = \int d^4y\, e^{-i(q+zp)\cdot y} \langle 0| \Phi_v^\dagger(0, -\infty)\; \psi(0)\, \bar{\psi}(y)\; \Phi_v(y, -\infty)|0\rangle \otimes V^{-1}\,. \tag{12}$$

We have defined the $v^\mu = \delta^\mu_+$ light-cone vector in the direction of motion of the incoming parton and $\bar{v}^\mu$ as the opposite $(-)$ direction. We have also introduced the Wilson line operator

$$\Phi_v(x + tv, -\infty) = P \exp\left[-ig_s \int_{-\infty}^t ds\, v^\mu\, A_\mu(x + sv)\right]\,. \tag{13}$$

The perturbative expansion of the Wilson line operator generates the Feynman rules for the eikonal interaction between a jet of fast moving partons in the v^μ direction and the soft gluons. The presence of the V^{-1} factor in the jet definitions removes any purely soft contributions from these functions. In a perturbative calculation of the jets, this corresponds to the prescription of applying soft subtractions in the contributing diagrams order by order.

In this operator language we can see that the factorization formula Eq. (9) comes from a rearrangement of the fields that define the currents to which the DIS probe couples (in our case the electromagnetic current $j^\mu = \bar{\psi}\gamma^\mu\psi$). Indeed, by "wiring" one of the fermion fields to the initial state and the other to the final state we obtain

$$P_r^{\mu\nu} W_{\mu\nu} = P_r^{\mu\nu} \mathrm{FT}_q^{(4)} \langle p|\bar{\psi}(0)\gamma_\mu\psi(0)\bar{\psi}(y)\gamma_\nu\psi(y)|p\rangle \approx \mathrm{FT}_{xp}^{(1)} \langle p|\bar{\psi}(0)\psi(y^-)|p\rangle \otimes \mathrm{FT}_{q+xp}^{(4)} P_r^{\mu\nu}\gamma_\mu \langle 0|\psi(0)\bar{\psi}(y)|0\rangle\gamma_\nu\,, \quad (14)$$

where we have introduced $\mathrm{FT}_{xp}^{(1)}$ and $\mathrm{FT}_q^{(4)}$ as shorthands for the one- and four-dimensional Fourier transforms. In this, leading jet, approximation, the F_r, $r = 2, L$, DIS structure functions are written as convolutions of two jets, which are identified as the initial and final state jets defined in Eqs (11,12). Note that this procedure would give a vanishing result for F_L, because the projection of the final state matrix element with P_L, Eq. (8), is zero by the Dirac equation of motion for the on-shell incoming quark. F_L vanishes in the leading-jet approximation, and the coefficient functions of F_L are less singular in the small parameter $1 - x$ relative to the ones of F_2, as we noted in the introduction. The operator formalism indicates how it can be expanded to capture such terms that are suppressed by powers of $1 - x$. To this end we introduce higher dimensional operators with the quantum numbers of the electromagnetic current and rearrange their fields between initial and final states. Let $O_i^\mu(y) = \bar{\psi}(y)\Xi_i^\mu(y)$ be an operator of dimension higher than 3. Then, as in Eq. (14), we obtain a new contribution

$$\begin{aligned} P_r^{\mu\nu} W_{\mu\nu} &= P_r^{\mu\nu}\, |H_L(Q^2)|^2\, \mathrm{FT}_q^{(4)} \langle p|\bar{\psi}(0)\Xi_{\mu,i}(0)\, \Xi_{\nu,i}^\dagger(y)\psi(y)|p\rangle \\ &\approx |H_L(Q^2)|^2\, \mathrm{FT}_{xp}^{(1)} \langle p|\bar{\psi}(0)\psi(y^-)|p\rangle \otimes \mathrm{FT}_{q+xp}^{(4)} P_r^{\mu\nu} \langle 0|\Xi_{\mu,i}(0)\Xi_{\nu,i}^\dagger(y)|0\rangle\,. \end{aligned} \quad (15)$$

with $H_L(Q^2)$ a new coefficient function. The second factor in the convolution product above is a *new* jet function. At each mass dimension for the operator O_i^μ we generate a class of jet functions whose contributions are suppressed by one factor of $1 - x$ relative to those of lower dimension level. Therefore, an expansion in powers of $1 - x$ reduces to dimensional counting, subject to the equations of motion of the incoming and outgoing partons. This is the non-local (bilocal) operator product expansion near the elastic limit [4]. In the language of effective field theories, we have "integrated out" the dynamics between the scales $(1 - x)Q^2$ and Q^2, which generates in a new operators O_i in an effective Hamiltonian for the still-perturbative dynamics at the intermediate scale.

The bilocal OPE in the elastic limit is a method of capturing corrections of order $(1 - x)^n$ in the final state interactions. This expansion is still leading twist in Q^2. This may be understood by observing that, if the dimension of the effective operator O_i^μ is $3 + n$, then a field of dimension $3/2 + n$ is emitted into the final state and the phase space integral scales as $(1 - x)^n Q^{2n}$. It is therefore the phase space integral that absorbs the factor $1/Q^{2n}$ associated with the coefficient functions for the higher dimensional operators, leaving the factor $(1 - x)^n$, which generates an expansion in $1 - x$.

Let us apply the above procedure to F_L. We seek to define a new jet function that captures the leading power contributions in $1 - x$. The dimension 4 operators with the quantum numbers of the current are

$$O_{2a}^\mu = \bar{v}^\mu \bar{\psi}\not{D}_\perp\psi\,, \quad O_{2b}^\mu = v^\mu\bar{\psi}\not{D}_\perp\psi\,, \quad O_{2c}^\mu = \bar{\psi}D_\nu\sigma_\perp^{\mu\nu}\psi\,, \quad O_{2d}^\mu = \bar{\psi}D^\mu\psi\,. \quad (16)$$

It is easily seen that O_{2b} and O_{2c} have vanishing projection when contracted with $P_L^{\mu\nu}$. They do not contribute to F_L but can contribute to F_2 at the subleading level. O_{2d} has a longitudinal projection that is proportional to $\bar{\psi}(v\cdot \overleftarrow{D})\psi$. After integration by parts this term vanishes by the equation of motion of ψ at $x = 1$. We therefore conclude that the only operator of dimension 4 that will contribute to F_L at the leading level is O_{2a}, with longitudinal projection

$$v_\mu O_{2a}^\mu = \bar{\psi}\, \overleftarrow{\not{D}}_\perp\, \psi\,. \quad (17)$$

This results in the jet function for F_L

$$J'((1-z)Q^2) = \left(\frac{1}{4\pi}\,\frac{8x^2}{Q^2}\right)\, \mathrm{FT}_{q+zp}^{(4)}\, \langle 0|\Phi_v^\dagger(0,-\infty)\,\not{D}_\perp\psi(0)\;\bar{\psi}(y)\,\overleftarrow{\not{D}}_\perp\;\; \Phi_v(y,-\infty)|0\rangle \otimes V^{-1}\,, \quad (18)$$

where we have restored the Wilson lines for manifest gauge invariance and have divided out any purely soft contributions. The resulting factorization formula for F_L is similar to the one for F_2.

$$F_L(x,Q^2) = |H_2(Q^2)|^2 \int_x^1 dx' \int_0^{x'-x} dw\ J'\left((x'-x-w)Q^2\right)\ V(w)\ \phi(x')\ \Big(1+\mathcal{O}(1-x)\Big)\,. \tag{19}$$

In the F_L resummation formula the anomalous dimension of J' will appear.

III. BEYOND THE EIKONAL APPROXIMATION - RESUMMATION

The factorization formula for F_L at large Bjorken x was derived via the non-local OPE near the elastic limit in the previous section. It is however instructive also to study the problem from the point of view of perturbation theory diagrams. This approach yields the Feynman rules for computing the anomalous dimensions of the higher jet functions.

The starting point of the diagrammatic approach is the observation that both F_2 and F_L are different projections of the *same* hadronic tensor $W^{\mu\nu}$. Any non-analytic terms in their expansion in massless perturbation theory arise from well known configurations in loop momentum space, the Landau pinch singular surfaces of $W^{\mu\nu}$. We can therefore use the infrared power counting and the jet-soft factorization techniques on which general QCD factorization theorems are based (for a review see Ref. [6]). Given the light-cone direction vectors v^μ and $\bar{v}^\mu$, normalized as $v\cdot\bar{v}=1$, the momenta of the incoming quark and the outgoing jet p and $\bar{p}$ respectively are

$$p^\mu = \frac{Q}{\sqrt{2}}v^\mu\,,\qquad \bar{p}^\mu = (1-x)\frac{Q}{\sqrt{2}}v^\mu + \frac{Q}{\sqrt{2}x}\bar{v}^\mu\,. \tag{20}$$

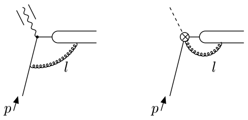

FIG. 1. The factorization of the $\bar{v}$-collinear gluon (l) from the hard subgraph generates the effective vertex $\otimes$. Parallel lines around the photon denote contraction with p^μ.

Consider the configuration shown in Fig. 1, in which the depicted gluon belongs to the $\bar{v}$-jet. The fermion line carrying momentum $p-l$ is far off-shell and belongs to the hard scattering subdiagram H_L. If, however, we were to factorize the top collinear gluon by eikonalizing the fermion propagator, then this, in general leading, IR contribution would vanish in F_L because of the contraction with p^μ from $P^{\mu\nu}$ at the current vertex. Note also in the case where the gluon is soft, the leading eikonal contribution is similarly projected out. Therefore, the expansion will start with terms that are subleading in the usual eikonal approximation. Expanding the diagram in the gluon's virtuality we obtain

$$g_s\frac{1}{-2p\cdot l+l^2}\ \not{p}(\not{p}-\not{l})\gamma^\lambda \approx g_s\left(\gamma_\perp^\lambda - l_\perp\cdot\gamma_\perp\frac{v^\lambda}{v\cdot l}\right)\equiv O^\lambda(l)\,, \tag{21}$$

where we have dropped terms vanishing by the incoming quark's equation of motion. An effective vertex to $\mathcal{O}(g_s)$ has emerged. This vertex, unlike the usual eikonal ones, probes the transverse momentum of the struck quark. It satisfies the following algebaic properties, that are useful in diagrammatic calculations:

$$O^\lambda(l)l_\lambda = 0,\quad O^\lambda(l)v_\lambda = 0,\quad O^\lambda(l)\bar{v}_\lambda = -g_s\ \not{l}_\perp\frac{1}{v\cdot l}\,,\quad O^\lambda(l)O_\lambda(l') = -2g_s^2,\quad O^\lambda(l)...O_\lambda(l') = g_s^2\gamma_\perp^\lambda...\gamma_{\perp\lambda}\,. \tag{22}$$

Moreover, it is noted that the vertex O^λ is the $\mathcal{O}(g_s)$ expansion in momentum space of the operator $\Phi_v(0,-\infty)\not{D}_\perp\psi(0)$, i.e. half of the bilocal operator that defines J', Eq. (18). It can be shown by induction that the above construction generalizes to any number of collinear gluons [7]. The factorization of the final state collinear partons from the initial state jet defines a final state jet function exactly as in Eq. (18).

Resummation may be thought of a consequence of factorization. [5] Once an observable is written as a product of effective operators, then the UV renormalization of these operators generates the evolution of the coefficient functions of the observable. Convolutions become simple products in moment space where the Mellin transform of F_L, Eq. (6), can be written as

$$\tilde{F}_L(N,Q^2,\epsilon) = \left| H_L\left(\frac{(p\cdot\bar{v})^2}{\mu^2},\frac{(\bar{p}\cdot v)^2}{\mu^2},\alpha_s(\mu^2)\right)\right|^2 \times\frac{1}{N}\tilde{J}'\left(\frac{Q^2}{N\mu^2},\frac{(\bar{p}\cdot v)^2}{\mu^2},\alpha_s(\mu^2)\right)\tilde{V}\left(\frac{Q^2}{N^2\mu^2},\alpha_s(\mu^2)\right)\tilde{\phi}\left(\frac{Q^2}{N^2\mu^2},\frac{(p\cdot\bar{v})^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right). \tag{23}$$

Here, the factor of $1/N$ has been exhibited explicitly,

$$\frac{1}{N}\tilde{J}'\left(\frac{Q^2}{N\mu^2},\frac{(\bar{p}\cdot v)^2}{\mu^2},\alpha_s(\mu^2)\right) = \int_0^1 dz\, z^{N-1}\, J'\left(\frac{(1-z)Q^2}{x\mu^2},\frac{(\bar{p}\cdot v)^2}{\mu^2},\alpha_s(\mu^2)\right). \tag{24}$$

The treatment of the J' evolution proceeds in the usual way [5]. The jet function satisfies two equations. The first is the Lorentz covariance equation, which describes how the jet varies in response to changes in the light-cone direction with respect to which it has been defined, and the second is the jet's own renormalization group equation,

$$\frac{\partial}{\partial\ln(\bar{p}\cdot v)^2}\ln\tilde{J}'\left(\frac{Q^2}{N\mu^2},\frac{(\bar{p}\cdot v)^2}{\mu^2},\alpha_s(\mu^2)\right) = K\left(\frac{Q^2}{N\mu^2},\alpha_s(\mu^2)\right) + G\left(\frac{(\bar{p}\cdot v)^2}{\mu^2},\alpha_s(\mu^2)\right), \tag{25}$$

$$\frac{d}{d\ln\mu^2}\ln\tilde{J}'\left(\frac{Q^2}{N\mu^2},\frac{(\bar{p}\cdot v)^2}{\mu^2},\alpha_s(\mu^2)\right) = -\frac{1}{2}\gamma_{J'}(\alpha_s(\mu^2)). \tag{26}$$

The right-hand side of the covariance equation is renormalization group invariant and the K factor renormalizes as

$$\frac{d}{d\ln\mu^2}K\left(\frac{Q^2}{N\mu^2},\alpha_s(\mu^2)\right) = -\frac{1}{2}\gamma_K(\alpha_s(\mu^2)), \tag{27}$$

where γ_K is the well known cusp anomalous dimension in $\overline{\mathrm{MS}}$ scheme,

$$\gamma_K(\alpha_s) = \frac{\alpha_s}{\pi}C_F + \left(\frac{\alpha_s}{\pi}\right)^2\left[C_F C_A\left(\frac{67}{36}-\frac{\pi^2}{12}\right) - \frac{5}{18}C_F N_f\right] + \mathcal{O}(\alpha_s^3). \tag{28}$$

Eqs (25,26) can be solved simultaneously subject to the initial condition

$$\tilde{J}'(1,1,\alpha_s(Q^2)) = C_F\frac{\alpha_s(Q^2)}{\pi} + \mathcal{O}(\alpha_s^2(Q^2)), \tag{29}$$

from the Mellin transform, Eq. (24), of J' to lowest order $\mathcal{O}(\alpha_s)$. To this order, there is only one contributing diagram, depicted in Fig. 2.

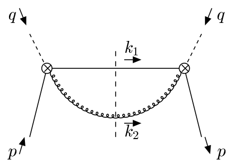

FIG. 2. The lowest order contribution to the jet function J'.

Substituting the solution of the jet equations (25) and (26) into the factorization formula (23), we obtain the resummed expression

$$\tilde{F}_L(N,Q^2,\epsilon) = C_F \frac{\alpha_s(Q^2)}{\pi N} (\tilde{V}\tilde{\phi})(\mu^2 = Q^2/N, \epsilon)$$
$$\times \exp\left[\frac{1}{2}\int_{Q^2/N}^{Q^2} \frac{d\mu^2}{\mu^2}\left(\ln N\, \gamma_K(\alpha_s(\mu^2)) + \gamma_{J'}(\alpha_s(\mu^2)) + 2\beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\ln \tilde{J}'(1,1,\alpha_s(\mu^2))\right)\right] + \mathcal{O}\left(\frac{\ln^0 N}{N}\right). \qquad (30)$$

Apart from the jet J', N dependence is also found from in the factor $\tilde{V}\tilde{\phi}$ in Eq. (23). This dependence, however, cancels in the coefficient function C_L once we divide $\tilde{F}_L$ by the Mellin transform of the parton distribution function $\tilde{f}$, Eq. (1). In Eq. (30), the contributions coming from the UV renormalization of the jet function J' are contained in $\gamma_{J'}$ and are kept distinct from the renormalization of the coupling, contained in the β-function term of the exponent. This distinction is important for the bookkeeping of the renormalization terms in the perturbative calculation of the anomalous dimensions.

In summary, we have shown that the extension of the bilocal OPE to the case of F_L modifies the Sudakov resummation formula essentially only through the introduction of the anomalous dimension of the new jet function J'. The cusp anomalous dimension γ_K contains all universal soft radiation effects.

IV. THE JET ANOMALOUS DIMENSION TO $\mathcal{O}(ALPHA_S)$

Inspecting the resummation formula, Eq. (30), we see that the only new ingredient is the anomalous dimension $\gamma_{J'}$. In this section, we outline its calculation to $\mathcal{O}(\alpha_s)$. Because F_L itself and J' start at $\mathcal{O}(\alpha_s)$, $F_L^{(1)} = J'^{(1)} = \bar{C}_L^{(1)} = C_F x$, we need to compute J' to $\mathcal{O}(\alpha_s^2)$. Using the rules for the effective operator O^λ of the previous section we identify the second order diagrams shown in Fig. 3.

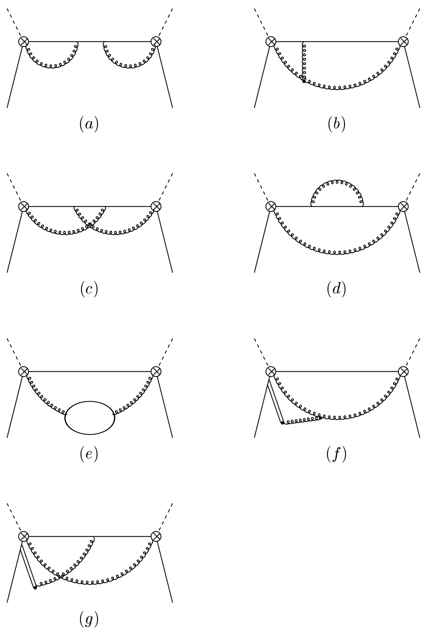

FIG. 3. The $\mathcal{O}(\alpha_s^2)$ diagrams for the jet function J'.

First we compute both the UV and IR singular parts of the diagrams. Then we perform an IR subtraction as dictated by Eq. (18). The infrared part to be subtracted out is $J'^{(1)} \otimes V^{(1)}$ and we note that the soft function V is the same for F_L and F_2. After the subtraction, only UV simple poles survive and from these we construct as usual the anomalous dimension $\gamma_{J'}$. The result is

$$\gamma_{J'}(\alpha_s) = \frac{\alpha_s}{\pi}\left[\frac{9}{2}C_F - 2C_A - 4\zeta(2)\left(C_F - \frac{C_A}{2}\right)\right] + \mathcal{O}(\alpha_s^2). \tag{31}$$

Comparison of the resummation with the fixed order results listed in the introduction is made by first writing the resummation prediction for the coefficient function to $\mathcal{O}(\alpha_s^2)$

$$\tilde{C}_L^{(2)} = \frac{C_F}{2N}\left[\gamma_K^{(1)} \ln^2\frac{N}{N_0} - \left(\gamma_{J'}^{(1)} - \frac{1}{2}\beta_1\right)\ln\frac{N}{N_0}\right] \tag{32}$$

with $N_0 = e^{-\gamma_E}$. Upon substituting for the anomalous dimensions γ_K and $\gamma_{J'}$ from Eqs. (28, 31) we obtain the Mellin transform of the fixed order result in Eq. (4).

V. SUMMARY

We have shown how the light-cone expansion for hard QCD processes near the elastic limit can be extended into a general bilocal OPE that captures power terms suppressed in $1-x$. This operator expansion is based on purely dimensional arguments, but it can also be derived from the analysis of the perturbative diagrams in the infrared limit. The result is a Sudakov resummation formula for the observable, in which the anomalous dimensions of the higher dimensional jet functions enter the exponent. We have analyzed the DIS longitudinal structure function at large Bjorken x as a concrete example of this operator expansion near the elastic limit. The anomalous dimension of the F_L final state jet was computed to first order in α_s, and the resummation formula for the coefficient function was shown to agree with the fixed order calculations.

We emphasize that our analysis is leading twist in Q^2 and is consistent with the usual light-cone expansion. It resums terms that are of order $\ln^k N/N$, $k>0$. This approach can be extended to the case of F_2, where such terms are present, but are subleading relative to order $\ln^k N$, $k \geq 0$ terms. Indeed we expect that the operator expansion presented here is relevant for all hard QCD processes in the quasi-elastic limit.

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