Light Hadron Spectrum and Heavy-light Decay Constants from the MILC Collaboration

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The light hadron spectrum in the continuum limit with staggered quarks has recently been computed in both the quenched approximation and with two flavors of dynamical quarks. Over a range of quark mass, a significant difference between the quenched and dynamical theories is observed for the nucleon to vector meson mass ratio. Decay constants for the B and B_s mesons are crucial for the accurate determination of the CKM mixing matrix. We have calculated several decay constants in the continuum limit within the quenched approximation. In addition, we have used two gauge couplings and a variety of dynamical staggered quark masses to address the effects of full QCD dynamics for the decay constants.

I. LIGHT-HADRON SPECTRUM

One of the important long term goals of lattice QCD has been an accurate calculation of the spectrum. Not only is the spectrum of interest in its own right, it is necessary to determine the quark masses. These masses are parameters within the Standard Model that must be determined from experiment. However, for any more fundamental theory that wishes to explain the Standard Model, these become calculable consequences or tests of the more fundamental theory. An accurate calculation of the spectrum would also give us confidence that we can calculate other quantities, such as the decay constants that are discussed below, and many additional hadronic matrix elements that have important phenomenological consequences.

Because lattice calculations cannot be done with infinite volume, zero lattice spacing or physically light quark masses, good control of potential systematic errors is essential. Due to limitations of space, we will not deal here with each of the potential sources of error. Rather, we shall concentrate on the quark mass dependence, or chiral extrapolation as is it often called. This has proven to be the source of error that requires the most care.

We have studied the spectrum with dynamical quarks and in the quenched approximation (*i.e.*, the approximation in which virtual quark anti-quark pairs are neglected). In the quenched approximation using Kogut-Susskind or staggered quarks, we find $m_N/m_\rho = 1.254 \pm 0.018 \pm 0.028$. (The experimental value is 1.22.)

For two flavors of dynamical quarks, we find that m_N/m_ρ is greater than for the quenched approximation over a wide range of quark mass. We shall not quote a value of m_N/m_ρ for physical quark mass because we have not yet gone close enough to the chiral limit for our two weakest couplings. For each value of the lattice spacing, we have studied at least five quark masses. Additional details may be found in Refs. [1]– [3].

To control systematic errors, we must have a large enough volume and be able to extrapolate in quark mass and lattice spacing. We like to put our system in a box that is at least 2.5 fm on a side. (We use periodic boundary conditions in space.) After we do the chiral extrapolation at each lattice spacing, it is easy to extrapolate in lattice spacing. The quark mass, or chiral extrapolation has proven to be difficult. Lattice calculations cannot be done with physical quark masses because the condition number of the Dirac matrix increases as the quark mass is reduced. This makes solving for the quark propagators too expensive. (This problem is worse for dynamical quarks.) In addition, the quarks are more sensitive to fluctuations in the gluon fields, and the statistical errors grow as the quark mass is decreased.

Chiral perturbation theory (χPT) provides the theoretical basis for the chiral extrapolation [4]. However, in quenched chiral perturbation theory (QχPT) [5,6] there are new $m_q^{1/2}$ and $m_q \log m_q$ terms due to η' loops that don't appear in ordinary χPT. We tried a fairly conventional approach of attempting to fit our masses using twelve different functions that contain terms from both χPT and QχPT. Some fits with $m_q^{1/2}$ were good, but the coefficient of that term had the opposite sign from the prediction of QχPT. (Further details may be found in Ref. [3].)

Due to flavor symmetry breaking for staggered quarks, the flavor singlet pseudoscalar that appears in QχPT, does not actually have a mass proportional to $m_q^{1/2}$. Thus, it seems more appropriate to express this term using the non-Goldstone pion mass. We define for fixed λ_N:

$$m'_N \equiv (m_N + \lambda_N m_{\pi_2}) \frac{m_N^{\rm phys}}{m_N^{\rm phys} + \lambda_N m_\pi^{\rm phys}} \tag{1}$$

with a similar expression for the ρ. We then fit m'_N and m'_ρ as in ordinary χPT for $0 \le \lambda_N \le \lambda_\rho \le 0.4$, the range expected from QχPT. The variation of our extrapolated value with λ_N and λ_ρ then determines our systematic error [3]. The factor in m'_N that depends on the physical nucleon and pion masses and λ_N is designed so that if the lattice nucleon and π_2 masses approach their physical values, then m'_N/m'_ρ will equal m_N/m_ρ. With this approach we find $m_N/m_\rho = 1.254 \pm 0.018 \pm 0.028$, where the first error is statistical and the second takes into account the variation from varying $\lambda_{N,\rho}$ and the way the continuum extrapolation is done. Adding the errors in quadrature, we see that our central value for this quenched calculation is just about one standard deviation high, and the error is just under 3%.

For the dynamical quark results, we don't have to worry about the special terms of QχPT. We fit N and ρ masses to the forms $M + am_q + bm_q^2$ plus either $cm_q^{3/2}$ or $cm_q^2 \log m_q$.

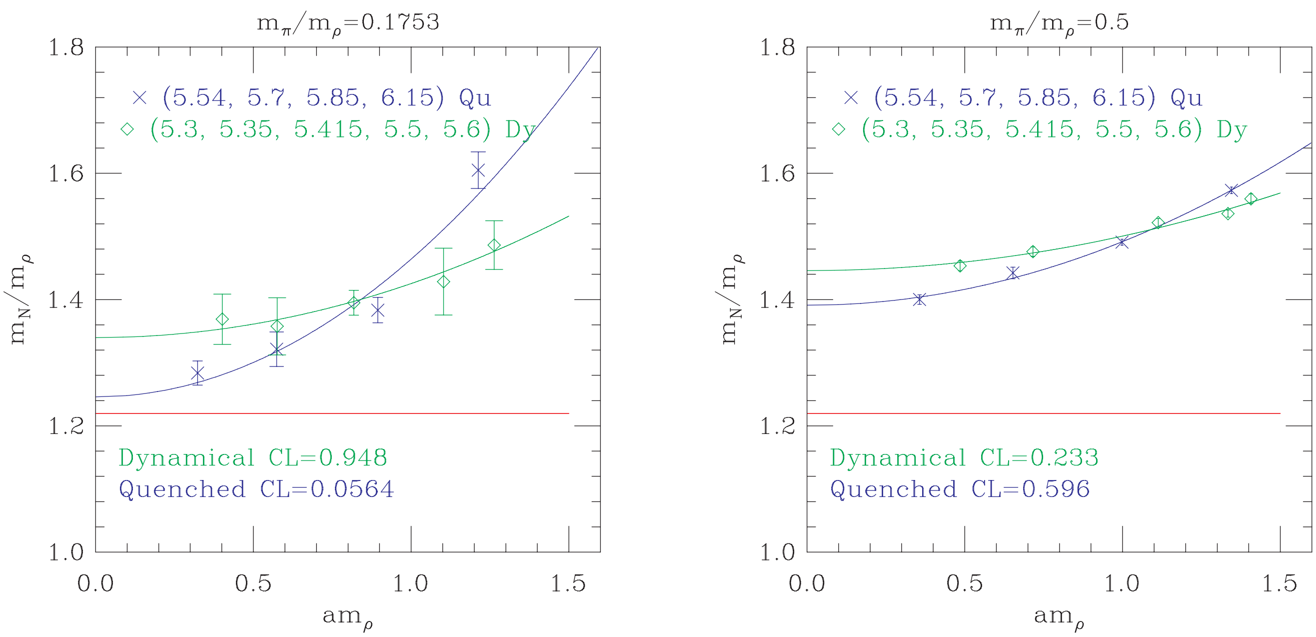

FIG. 1. The continuum extrapolation for the nucleon to rho mass ratio. Both quenched and dynamical results are shown. The continuum fit includes a term quadratic in am_ρ, and the confidence level of the fit is shown at the bottom of the figure.

At each coupling we can adjust the quark mass to get any desired value of m_π/m_ρ. In this way, we can take the continuum limit with any desired quark mass. In Fig. 1, we consider the continuum limit for $m_\pi/m_\rho = 0.1753$ (the physical value) and 0.5 (a heavier value where we hardly need any extrapolation in quark mass). The horizontal coordinate is the ρ mass for the quark mass that gives the chosen value of m_π/m_ρ. We see a clear difference between the quenched and dynamical results. In these graphs we use conventional χPT fits for both the quenched and dynamical data. Each graph displays the confidence level for the continuum extrapolation and a horizontal line is drawn at the experimental value of m_N/m_ρ.

We carry out this analysis for additional values of m_π/m_ρ between 0.3 and 0.65. Plotting the value in the continuum limit and its error, we find the Edinburgh curves for quenched and dynamical quarks shown in Fig. 2. Over a wide range of quark mass, there is a significant difference between the two results. For instance, at $m_\pi/m_\rho = 0.55$ (where we do not require any chiral extrapolation) the difference is 0.041 ± 0.007. The octagon to the upper right is the heavy quark limit and the one to the lower left is the experimental result. The point plotted with a burst (slightly displaced to the left) is the result obtained above using the method consistent with QχPT. If we vary the chiral fit for the dynamical results by substituting $m_q^2 \log m_q$ for $m_q^{3/2}$, the curve is shifted less than one standard deviation upward at small quark mass. There is very little change near $m_\pi/m_\rho = 0.5$.

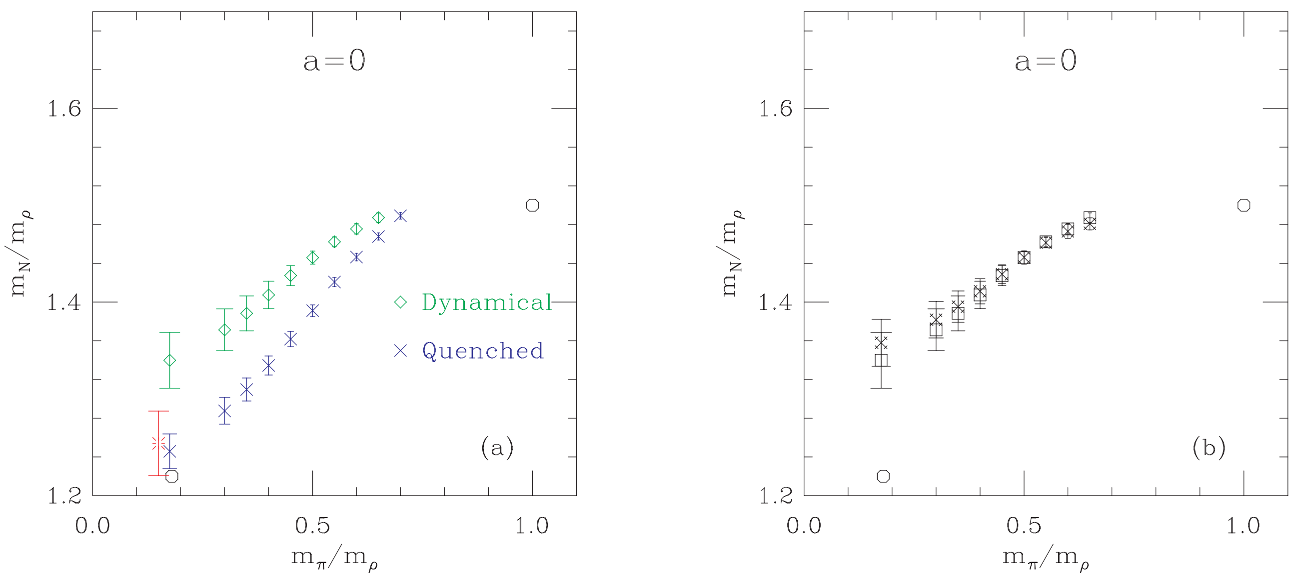

FIG. 2. The Edinburgh curves in the continuum limit for Kogut-Susskind quarks. (a) Both the quenched approximation and $N_f = 2$ dynamical quarks are shown. The chiral extrapolations for nucleon and rho are fits to the function $M + am_q + bm_q^{3/2} + cm_q^2$. The burst, slightly displaced to the left is from our fits that include a π_2 term for consistency with QχPT. (b) For dynamical quarks we compare two different chiral extrapolations for the nucleon and rho. The squares are fits to $M + am_q + bm_q^{3/2} + cm_q^2$ and the fancy crosses are fits to $M + am_q + bm_q^2 \log m_q + cm_q^2$.

Since the quenched result at the physical value of m_π/m_ρ is closer to the experimental result than our result with dynamical quarks, we are left in quite a state of surprise. How can this be? Possible explanations include:

First, for the dynamical results, we have not gotten as close to the chiral limit as we have for the quenched case. We are, therefore, extrapolating further in that case. The errors at light quark mass are clearly larger than in the quenched case. Thus, we don't feel we have as much control over potential systematic errors as we would like to have.

Second, for the dynamical results, we take no special measures to deal with the issue of ρ decay. We may need a full multi-channel analysis of the ρ and two π states to correctly determine the ρ mass. Of course, this same concern would be relevant for all previous lattice calculations.

Third, the real world has a dynamical strange quark. Perhaps it plays a more significant role in the light hadron spectrum than we might have imagined.

Now that we can do calculations of the light quark spectrum with errors of less than one percent on individual points and about one to three percent even after extrapolations, we can really start to test our understanding of systematic effects in a way that was not possible a few years ago.

II. HEAVY-LIGHT DECAY CONSTANTS

B-meson decay constants are needed to understand $B\bar{B}$ ($B_s\bar{B}_s$) mixing and extract V_{td} (V_{ts}) from experiment. Since such mixing has recently been seen, and will be intensely studied in the next few years at B factories, it is crucial for the lattice community to calculate these decay constants with well controlled systematic errors. Further, D_s decay has already been measured [7] and provides a good check of our calculations.

Our calculation of decay constants uses Wilson light valence quarks and either Wilson or static heavy valence quarks. We calculate their propagation in configurations that are either quenched or have $N_f = 2$ staggered dynamical quarks. Because we have a wider variety of quenched lattice spacings, our published results to date report a continuum limit for the quenched decay constants and use the dynamical configurations only to estimate the quenching error (by comparing with the quenched results at fixed lattice spacing). We have very recently finished analyzing enough dynamical $N_f = 2$ data to be able to extrapolate it to the continuum limit. Some preliminary results from that extrapolation are reported below. Those results, even when completed, will still not be "full QCD" because only two light flavors or virtual quarks have been included — we still need to add in virtual strange quark loops.

To calculate the decay constants, we must determine the quark masses and set a scale. We need m_l (a common quark mass for u and d quarks), m_s, (m_c for D mesons) and m_b. We set the scale from f_π. The ρ mass is another possibility, but in either case, we need a chiral extrapolation to set the scale. For each of our data sets we have three quark masses in the range $0.7m_s \lesssim m_q \lesssim 2.0m_s$. To carry out the chiral extrapolations, we use the kinetic mass m_2, rather than $1/\kappa$, as the independent variable. It gives us acceptable-to-good confidence level (CL) for linear fits of m_{Qq}, f_{Qq}, for all sets, where Q can be either a static or heavy quark and q represents a light quark. However, linear fits are not adequate for determination of κ_c or for the fits of f_π for the stronger couplings. For the central values, we use quadratic fits of m_π^2 *vs.* m_2, and linear fits of M_{Qq}, f_{Qq}, and f_π. To estimate the systematic error in the chiral extrapolations, we consider three additional combinations of chiral fits (two with additional quadratic fits, one with all linear fits), for a total of four. Additional details may be found in Ref. [9].

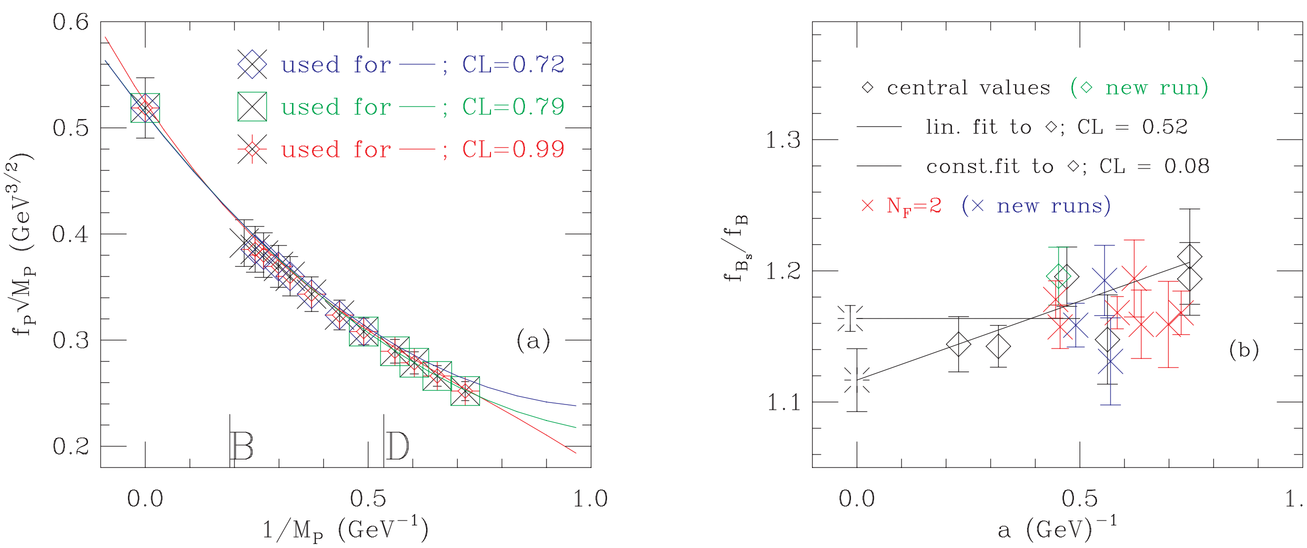

FIG. 3. (a) $f_P\sqrt{M_P}$ *vs.* $1/M_P$ for $6/g^2 = 6.52$, our smallest lattice spacing. (b) Continuum extrapolation of f_{B_s}/f_B. Both quenched and dynamical points are shown. The continuum extrapolations are for the quenched results.

The heavy valence quarks are treated using the $\sqrt{1-6\tilde{\kappa}}$ EKM norm and a shift to the kinetic mass ("$m_1 \rightarrow m_2$") [8]. After removing perturbative logs, the heavy-light and static-light results for $f_P\sqrt{M_P}$ are fit to a polynomial to $1/M_P$ and interpolated to the B or D (B_s or D_s) mass. Here P is a general heavy-light pseudoscalar with the light quark extrapolated to the u, d (s) mass, and arbitrary heavy quark mass. This is illustrated in Fig. 3a where three fits are shown. The three fits all include the static point and then either the entire range of displayed values, the values close to the D or the values between the D and B masses. On the horizontal axis vertical lines are drawn corresponding to the B and D masses. The scale is set by f_π and these results are from our weakest coupling quenched run, $6/g^2 = 6.52$.

Perturbative corrections are required to match the lattice decay constants to the continuum. For the heavy-light case, we use a mass-dependent perturbative renormalization (at one-loop) of the axial current that was calculated by Kuramashi [10]. We also use tadpole improved perturbation theory, which requires determination of a scale q^* characteristic of the particular process under consideration. A calculation gives $q^* = 2.32/a$ for (massless) Wilson quarks [12] and $q^* = 2.18/a$ for the static-light case [11]. We vary q^* over a wide range to estimate the systematic error.

Next, we extrapolate to the continuum limit. This is done in two different ways. (See Fig. 3b.) In the first way, we use a linear fit to the data from all quenched data sets. In the second way, we fit the data to a constant using results from the three quenched lattices with smallest lattice spacings ($\beta = 6.0, 6.3, 6.52$). In carrying out these fits, we take the errors in points at fixed lattice spacing to be the sum in quadrature of statistical errors and the systematic variations due to various choices of fitting intervals in t.

The largest sources of systematic error within the quenched approximation come from the chiral extrapolation, the continuum extrapolation and the perturbative corrections. These errors interact strongly with each other and cannot be estimated separately. Instead, we repeat the analysis 24 times (4 chiral fits $\times$ 2 continuum extrapolations $\times$ 3 q^* scale choices). This gives us our central value and 23 alternatives that are used to calculate the dispersion.

The final error to discuss is that due to quenching. In our published results, we take as an estimate of the error the difference between the weakest coupling $N_f = 2$ lattices and the quenched results interpolated (*via* the linear fit) to the same value of the lattice spacing. These results are summarized below. The first error is statistical, the second is the systematic error within the quenched approximation, and the third is an estimate of the error from quenching. As we have recently learned from the extrapolation of the $N_f = 2$ results to the continuum, the quenching error estimated as just described appears to be considerably smaller than the true quenching error. This is because the quenched and dynamical results have rather different behavior as a function of lattice spacing, and the difference increases as the continuum limit is approached.

$$\begin{aligned}
f_B &= 154 \pm 11 \,{}^{+27}_{-\ 7}\,{}^{+23}_{-\ 0} \quad \text{MeV} \\
f_{B_s} &= 169 \pm \ 9 \,{}^{+35}_{-8}\,{}^{+27}_{-\ 0} \quad \text{MeV} \\
f_D &= 191 \pm 10 \,{}^{+19}_{-10}\,{}^{+15}_{-\ 0} \quad \text{MeV} \\
f_{D_s} &= 208 \pm \ 8 \,{}^{+26}_{-\ 8}\,{}^{+17}_{-\ 0} \quad \text{MeV}
\end{aligned}$$

$$\begin{aligned}
f_{B_s}/f_B &= 1.12 \pm 0.02 \,{}^{+0.04}_{-0.03}\,{}^{+0.03}_{-0.03} \\
f_{D_s}/f_D &= 1.10 \pm 0.02 \,{}^{+0.04}_{-0.02}\,{}^{+0.02}_{-0.03} \\
f_B/f_{D_s} &= 0.75 \pm 0.03 \,{}^{+0.04}_{-0.02}\,{}^{+0.08}_{-0.00} \\
f_{B_s}/f_{D_s} &= 0.85 \pm 0.03 \,{}^{+0.05}_{-0.03}\,{}^{+0.05}_{-0.00}
\end{aligned}$$

In Fig. 4, we show some of our recent attempts to extrapolate to the continuum limit our results for f_B and f_{B_s}/f_B with two flavors of dynamical light quarks. We show two extrapolations: one based on linear fits and one where the result is fit to a constant. Assuming that one can trust the extrapolations all the way to $a = 0$, the presence of virtual quark loops has raised f_B by a large amount, while it has had a considerably smaller effect on the ratio f_{B_s}/f_B. Our preliminary results including two flavors of dynamical light quarks are then:

$$\begin{aligned}
f_B &= 210 \pm 3 \pm 26 \pm 28 \quad \text{MeV} \\
f_{B_s} &= 246 \pm 3 \pm 25 \pm 38 \quad \text{MeV} \\
f_{D_s} &= 263 \pm 2 \pm 17 \pm 28 \quad \text{MeV}
\end{aligned}$$

$$f_{B_s}/f_B = 1.17 \pm 0.02 \,{}^{+0.03}_{-0.04} \pm 0.03$$

Here the first errors are statistical and the second are systematic errors within the $N_f = 2$ computation. The third is a (very crude) estimate of the possible effects of introducing virtual strange quark loops. At this point it is found by taking 50% of the difference between the $N_f = 2$ and quenched results in the continuum limit.

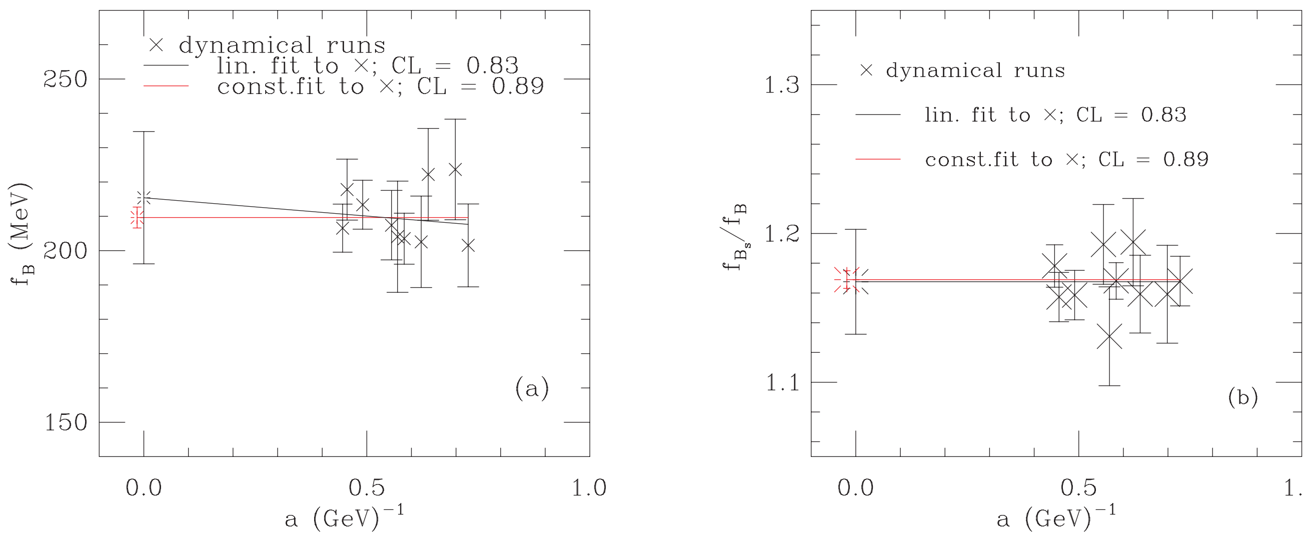

FIG. 4. The continuum extrapolation for f_B and f_{B_s}/f_B with dynamical quarks. The continuum fit is either to a constant or a linear function.

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