

FIG. 5. $\text{Re}A_0$ for quenched ensembles plotted against lattice spacing squared. The horizontal line shows the experimental result of $27.2 \cdot 10^{-8}$ GeV. Only statistical errors are shown.

IV. DIFFICULTIES WITH ε'

In addition to the difficulties mentioned above, estimation of ε' is further complicated by the issue of matching lattice and continuum operators. Usually such matching is done using lattice perturbation theory. For some operators important for ε' , the perturbation theory is unreliable. This is a serious obstacle for estimating ε' , especially considering that it is an extremely fragile quantity, dependent on several subtle cancellations. What is needed is a non-perturbative matching procedure, similar to the one described in Ref. [5].

As a temporary step, we have adopted a partially non-perturbative matching procedure, based on computing bilinear renormalization constants [1]. We can obtain only an approximate guide to the size of four-fermion operator matching coefficients, since a few *ad hoc* assumptions are made. If this procedure is true, we get a surprising result: $\varepsilon'/\varepsilon < 0$. This result and the validity of our assumptions should be verified by full non-perturbative operator matching.

V. CONCLUSION

We have computed MEs of all basis operators appropriate for predicting the magnitude of $\Delta I = 1/2$ amplitude enhancement and ε'/ε , on a number of gauge ensembles with reasonable statistical accuracy. A few systematic uncertainties still preclude a definite result for either of the two quantities. For $\Delta I = 1/2$ rule, the main uncertainty is due to unknown higher-order chiral corrections, while for ε'/ε there is an additional uncertainty due to the difficulties with operator matching. We observe that within this sizeable uncertainty, the prediction for $\text{Re}A_0/\text{Re}A_2$ ratio is of the same order as the experiment.

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- [1] D. Pekurovsky and G. Kilcup, hep-lat/9812019.
 - [2] A. Buras *et al.*, Nucl. Phys. **B 408** (1993) 209-285, hep-ph/9303284; A. Buras, hep-ph/9806471.
 - [3] L. Maiani, M. Testa, Phys. Lett. **B 245** (1990) 585.
 - [4] C. Bernard, T. Draper, A. Soni, H.D. Politzer and M. B. Wise, Phys. Rev. **D 32** (1985) 2343.
 - [5] A. Donini, V. Gimenez, G. Martinelli, G.C. Rossi, M. Talevi, M. Testa, A. Vladikas, Nucl. Phys. **B (Proc. Suppl.) 53** (1997) 883 (LATTICE '96), hep-lat/9608108; A. Donini, G. Martinelli, C.T. Sachrajda, M. Talevi, A. Vladikas, Phys. Lett. **B 360** (1995) 83, hep-lat/9508020.

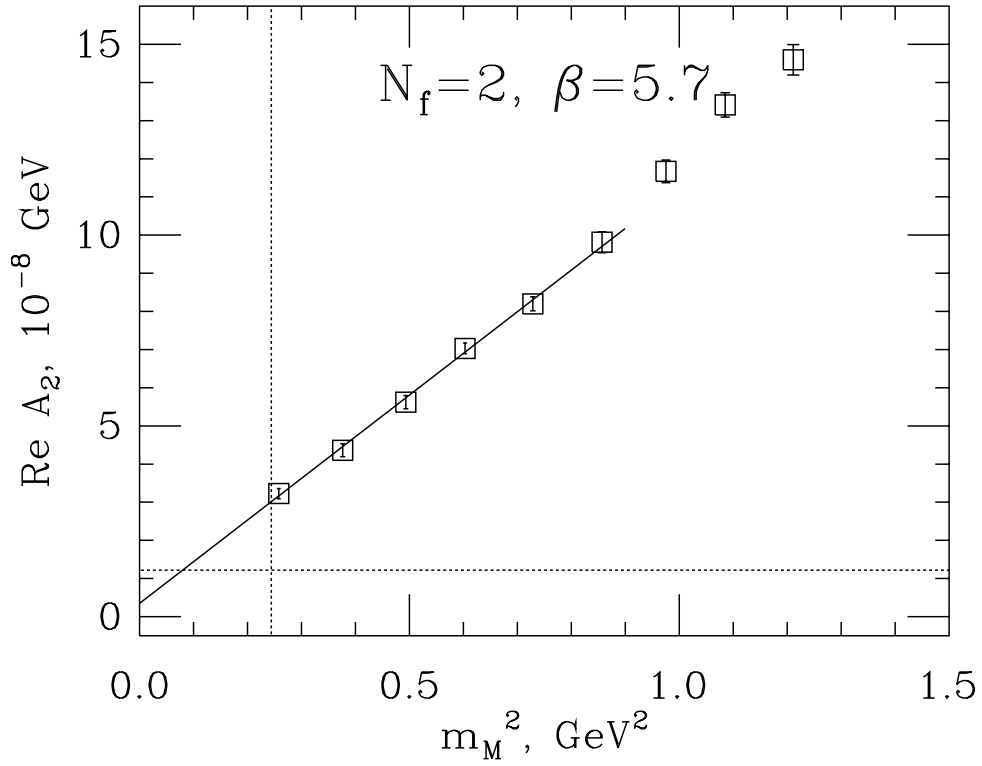


FIG. 3. $\text{Re}A_2$ for the dynamical ensemble. The horizontal line is the experimental value of 1.23 GeV.

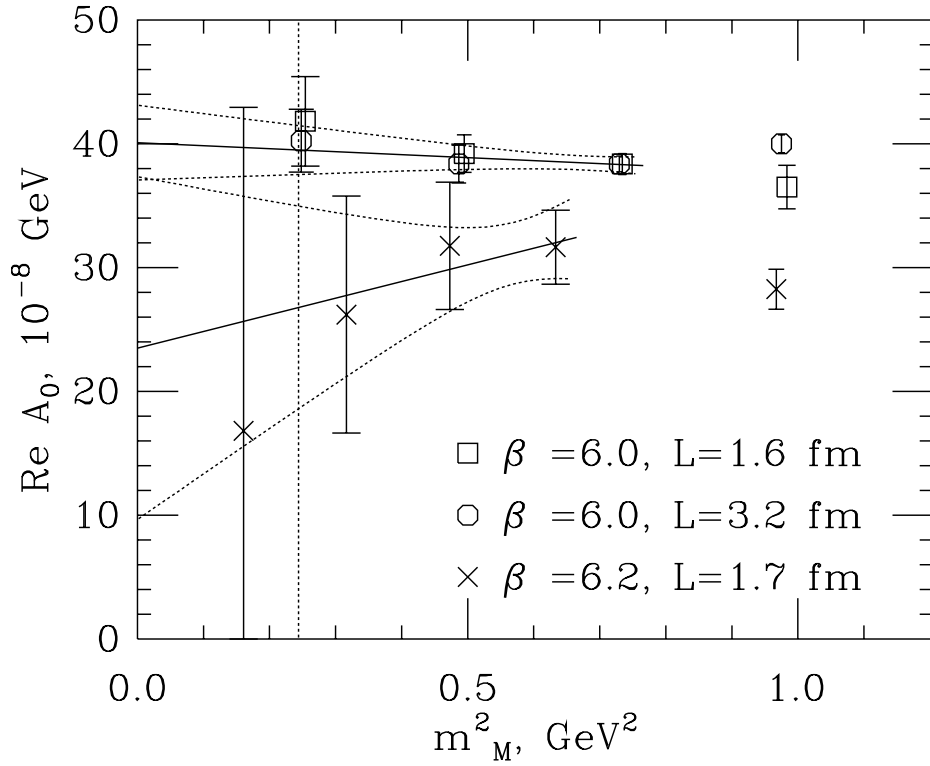


FIG. 4. $\text{Re}A_0$ for quenched ensembles plotted against the meson mass squared. The upper group of points is for ensembles Q_1 and Q_2 , while the lower group is for Q_3 . Only statistical errors are shown.

In Table I we show the simulation parameters. We are using a quenched $\beta = 6.0$ ensemble, and compare it with a quenched $\beta = 6.2$ ensemble to study cutoff dependence, with a $\beta = 5.7$ ensemble with two dynamical quark flavors (with comparable cutoff size) to check the magnitude of quenching effects, and with another quenched $\beta = 6.0$ ensemble to check the finite volume dependence.

Shown in Fig. 1 are the diagrams we are computing. The “eight” diagrams, along with the two-point function and “subtraction” diagrams, are relatively cheap to compute. In contrast, the “eye” and “annihilation” diagrams have been known in the past to be noisy and therefore difficult to analyze. In the present study we have gained enough statistics to compute them with reliable accuracy.

We use staggered fermions and gauge-invariant, tadpole-improved operators throughout the simulation. The masses of all light quarks are equal. Other details and exact expressions for various quantities can be found in our upcoming paper [1].

III. RESULTS OF THE SIMULATION: $\Delta I = 1/2$ RULE

The main results of this work are shown in Fig. 2. The dependence of isospin amplitude ratio on the kaon mass is quite dramatic. This is due to the behaviour of $\text{Re}A_2$ amplitude, shown in Fig. 3 (for dynamical ensemble). The $\text{Re}A_0$ amplitude is rather weakly dependent on the kaon mass (Fig. 4). Dependence of $\text{Re}A_0$ amplitude on the cutoff size is shown in Fig. 5. The finite volume dependence and the effect of quenching are found small compared to noise.

In order to compare the theoretical prediction for the amplitude ratio with experiment, it is necessary to specify a mass point for extrapolation in Fig. 2. If higher-order chiral relationships were used to obtain these amplitudes, they would provide a natural mass point. At this stage, all we can do is suggest that the mass point is somewhere between kaon and pion mass. This would give us a result approximately consistent with experiment, especially for the dynamical ensemble. Independent of the exact mass point, a significant enhancement of $\Delta I = 1/2$ over $\Delta I = 3/2$ transition is evident.

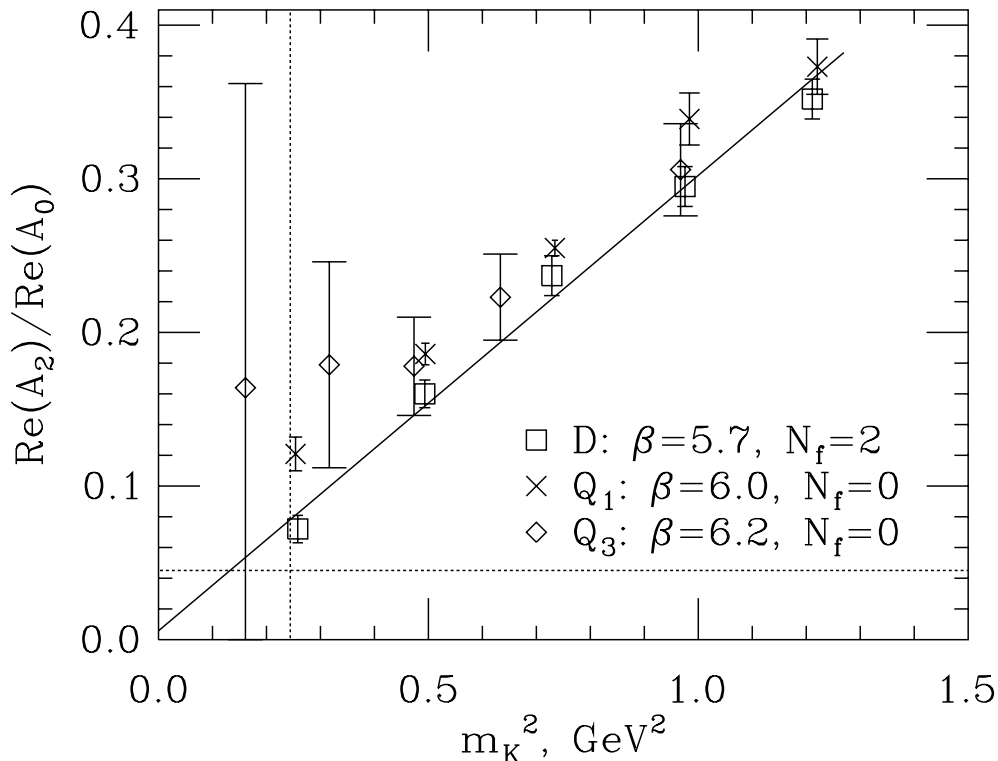


FIG. 2. The ratio $\text{Re}A_2/\text{Re}A_0$ versus the meson mass squared for quenched and dynamical ensembles. The dynamical ensemble data were used for the fit. The horizontal line shows the experimental value of $1/22$. The vertical line corresponds to the physical kaon mass. The error bars show only the statistical errors.

TABLE I. Lattice parameters

N_f	$\beta = \frac{6}{g^2}$	Lattice size	L, fm	Number of configurations	Quark masses used
0	6.0	$16^3 \times (32 \times 4)$	1.6	216	0.01 — 0.05
0	6.0	$32^3 \times (64 \times 2)$	3.2	26	0.01 — 0.05
0	6.2	$24^3 \times (48 \times 4)$	1.7	26	0.005 — 0.03
2	5.7	$16^3 \times (32 \times 4)$	1.6	83	0.01 — 0.05

For technical reasons, two-hadron states are very difficult to put on the lattice [3]. We compute $\langle \pi | O_i | K \rangle$ and $\langle 0 | O_i | K \rangle$ and use chiral perturbation theory to recover $\langle \pi \pi | O_i | K \rangle$ as follows [4]:

$$\langle \pi^+ \pi^- | O_i | K^0 \rangle = \langle \pi^+ | O_i - \alpha_i O_{sub} | K^+ \rangle \cdot \frac{m_K^2 - m_\pi^2}{(p_\pi \cdot p_K) f}, \quad (4)$$

where

$$\alpha_i = \frac{\langle 0 | O_i | K^0 \rangle}{\langle 0 | O_{sub} | K^0 \rangle}. \quad (5)$$

With staggered fermions, there is only one (lower-dimensional) operator that needs to be subtracted non-perturbatively in the manner shown above, namely

$$O_{sub} \equiv (m_d + m_s) \bar{s} d + (m_d - m_s) \bar{s} \gamma_5 d. \quad (6)$$

It should be kept in mind that this procedure is based on the lowest-order relationships in the chiral perturbation theory, and thus may be significantly biased. In particular, it ignores the final state interactions between pions among other higher-order corrections.

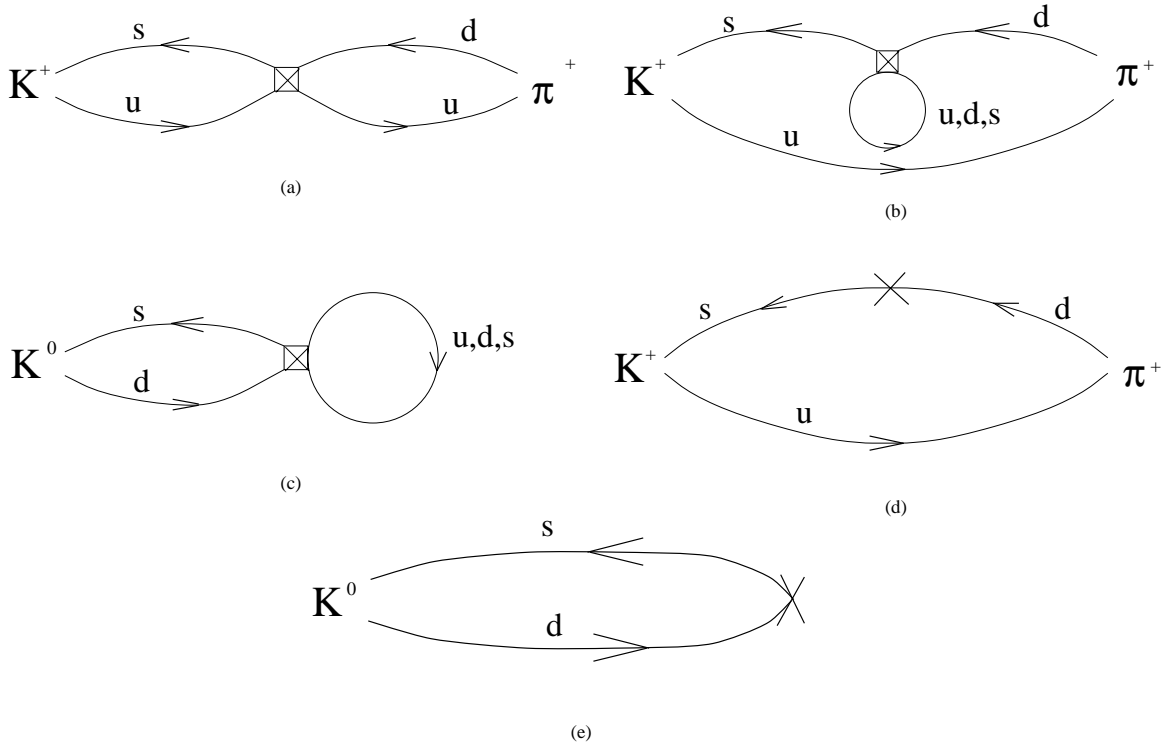


FIG. 1. Five diagrams types needed to be computed: (a) “Eight”; (b) “Eye”; (c) “Annihilation”; (d) “Subtraction”; (e) two-point function.

Lattice calculation of matrix elements relevant for $\Delta I = 1/2$ rule and ε'/ε .

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We have gained enough statistical precision to distinguish signal from noise in matrix elements of all operators relevant for the $\Delta I = 1/2$ rule in kaon decays and for the direct CP violation parameter ε' . We confirm significant enhancement of $\Delta I = 1/2$ transitions observed in experiments, although a few large systematic uncertainties remain in our predictions: higher-order chiral corrections and lattice spacing dependence. The estimate of ε'/ε is further complicated by the problem of matching lattice and continuum operators.

I. INTRODUCTION

One of the poorly understood effects in low-energy phenomenology is the so-called $\Delta I = 1/2$ rule. Namely, kaon decays proceed at much higher rate through the $\Delta I = 1/2$ channel than through the $\Delta I = 3/2$ one. In particular, for decays to two pions the following relationship between the amplitudes is experimentally observed:

$$\frac{A(K \rightarrow (\pi\pi)_{I=0})}{A(K \rightarrow (\pi\pi)_{I=2})} \equiv \frac{A_0}{A_2} = 22. \quad (1)$$

In the Standard Model, the electro-weak interactions and the short-distance part of strong interactions are not enough to explain this predominance of $\Delta I = 1/2$ transitions. The bulk of the ratio in Eq. 1 is attributed to the long-distance part of strong interactions. The long-distance effects are contained in the matrix elements (MEs) of the basis operators of the weak effective theory between hadron states. These MEs have remained mostly unknown due to their non-perturbative character. We compute them on the lattice with statistics more than sufficient to distinguish signal from noise for the $\Delta I = 1/2$ amplitude, thus allowing to check the prediction of the Standard Model and QCD against the experiment.

In a related effort, we address the direct CP-violating parameter ε' , defined in

$$\varepsilon' = \frac{\langle \pi^+ \pi^- | K_2 \rangle}{\langle \pi^+ \pi^- | K_1 \rangle}, \quad (2)$$

where K_1 (K_2) are CP-even (odd) strong interaction eigenstates. Our goal is to predict the value of ε' in the Standard Model in order to see if it is different from the Superweak theory and to compare it with experiment. As for the experimental situation, Fermilab's KTeV group has recently announced their latest result of $\text{Re}(\varepsilon'/\varepsilon) = (28 \pm 4.1) \cdot 10^{-4}$, which is consistent with the old result from CERN's NA48 experiment $((23 \pm 7) \cdot 10^{-4})$, although hardly consistent with the Fermilab's previous result of $(7.4 \pm 5.9) \cdot 10^{-4}$. The ε'/ε value will be even more refined soon by including more data from Fermilab and by complementing it with results from two other high-precision experiments at CERN and ϕ factory at Frascati. The theory is lagging behind these experiments, mostly due to the lack of knowledge of MEs containing long-distance dynamics.

II. SIMULATION DETAILS

We work within the effective theory obtained from the Standard Model by integrating out the W boson and t , b and c quarks [2]. The effective Hamiltonian is written in terms of linear combination of basis four-fermion operators:

$$H_W^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] O_i(\mu), \quad (3)$$

where z_i and y_i are Wilson coefficients (currently known at two-loop order) and $\tau \equiv -V_{td} V_{ts}^*/V_{ud} V_{us}^*$. In this work we seek to compute MEs $\langle \pi\pi | O_i | K \rangle$, which are necessary for estimation of both A_0 and A_2 amplitudes and ε' .