$|\Delta I| = 3/2$ Decays of Hyperons in Chiral Perturbation Theory

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We study the $|\Delta I| = 3/2$ amplitudes of the octet-hyperon decays $B \to B'\pi$ and of the decays $\Omega^- \to \Xi\pi$ in the context of heavy-baryon chiral perturbation theory. For the octet-hyperon decays, we investigate the theoretical uncertainty of the lowest-order predictions by calculating the leading nonanalytic corrections. We find that these corrections are within the expectations of naive power counting and, therefore, that this picture can be tested more accurately with improved measurements. For the Ω^- decays, we obtain at leading order two operators responsible for the decays which also contribute at one loop to the octet-hyperon decays. These one-loop contributions are sufficiently large to suggest that the measured ratio , $(\Omega^- \to \Xi^0\pi^-)/, (\Omega^- \to \Xi^-\pi^0) \approx 2.7$ may be too large.

I. INTRODUCTION

Nonleptonic decays of hyperons have been studied by various authors in the framework of chiral perturbation theory (χPT). For the hyperons belonging to the baryon octet, the decay modes are $\Sigma^+ \to n\pi^+$, $\Sigma^+ \to p\pi^0$, $\Sigma^- \to n\pi^-$, $\Lambda \to p\pi^-$, $\Lambda \to n\pi^0$, $\Xi^- \to \Lambda\pi^-$, and $\Xi^0 \to \Lambda\pi^0$. Calculations of the dominant $|\Delta I| = 1/2$ amplitudes of these decays have led to mixed results [1–6]. Specifically, the theory can give a good description of either the S-waves or the P-waves, but not both simultaneously. Now, while these amplitudes have been much studied in χPT, the same cannot be said of their $|\Delta I| = 3/2$ counterparts. In view of the situation in the $|\Delta I| = 1/2$ sector, it is instructive to carry out a similar analysis of the $|\Delta I| = 3/2$ amplitudes. Such an analysis has been done recently [7], and some of its results will be presented here.

In the baryon-decuplet sector, only the Ω^- hyperon is known to decay weakly. For $\Omega^- \to \Xi\pi$ decays, a purely $|\Delta I| = 1/2$ weak interaction would imply the ratio of decay rates , $(\Omega^- \to \Xi^0\pi^-)/, (\Omega^- \to \Xi^-\pi^0) = 2$. Instead, this ratio is measured to be approximately 2.7 [8], which seems to suggest that the $|\Delta I| = 1/2$ rule is violated in Ω^- decays [5]. This situation has recently been examined in some detail [9] using χPT. The result will also be presented here, for the couplings generating the $|\Delta I| = 3/2$ decays of the Ω^- also contribute to the octet-hyperon decays.

To apply χPT to interactions involving the lowest-lying mesons and baryons, we employ the heavy-baryon formalism [4,10]. In this approach, the theory has a consistent chiral expansion, and the octet and decuplet baryons in the effective chiral Lagrangian are described by velocity-dependent fields. We include the decuplet baryons in the Lagrangian because the octet-decuplet mass difference is small enough to make their effects significant on the low-energy theory [4,11].

II. $|\Delta I| = 3/2$ DECAYS OF OCTET HYPERONS

The leading-order chiral Lagrangian for the strong interactions is well known [4,10], and so we will discuss only the weak sector. Within the standard model, the $|\Delta S| = 1$, $|\Delta I| = 3/2$ weak transitions are induced by an effective Hamiltonian that transforms as $(27_{\rm L}, 1_{\rm R})$ under chiral rotations. At lowest order in χPT, the Lagrangian that describes such weak interactions of baryons and has the required transformation properties is [7,12]

$$\mathcal{L}^{\rm w} = \beta_{27} T_{ij,kl} \left(\xi \bar{B}_v \xi^\dagger\right)_{ki} \left(\xi B_v \xi^\dagger\right)_{lj} + \delta_{27} T_{ij,kl} \xi_{kd}\xi^\dagger_{bi} \xi_{le}\xi^\dagger_{cj} \left(\bar{T}^\mu_v\right)_{abc} \left(T_{v\mu}\right)_{ade} + {\rm h.c.} , \tag{1}$$

where β_{27} (δ_{27}) is the coupling constant for the baryon-octet (baryon-decuplet) sector, and $T_{ij,kl}$ is the tensor that project out the $|\Delta S| = 1$, $|\Delta I| = 3/2$ transitions (further details are given in Ref. [7]).

We now turn to the calculation of the amplitudes. In the heavy-baryon approach, the amplitude for the decay $B \to B'\pi$ can be written as [7]

$$i\mathcal{M}_{B \to B'\pi} = G_{\rm F} m_\pi^2 \, \bar{u}_{B'} \left(\mathcal{A}^{\rm (S)}_{BB'\pi} + 2k \cdot S_v \, \mathcal{A}^{\rm (P)}_{BB'\pi}\right) u_B , \tag{2}$$

where the superscripts refer to S- and P-wave contributions, the u's are baryon spinors, k is the outgoing four-momentum of the pion, and S_v is the velocity-dependent spin operator [10].

At tree level, $\mathcal{O}(1)$ in χPT, contributions to the amplitudes come from diagrams each with a weak vertex from $\mathcal{L}^{\mathrm{w}}$ in (1) and, for the P-waves, a vertex from the lowest-order strong Lagrangian. At next order in χPT, there are amplitudes of order m_s, the strange-quark mass, arising both from one-loop diagrams with leading-order vertices and from counterterms. Currently there is not enough experimental input to determine the value of the counterterms. For this reason, we follow the approach that has been used for the $|\Delta I| = 1/2$ amplitudes [1,3] and calculate only nonanalytic terms up to $\mathcal{O}(m_s \ln m_s)$. These terms are uniquely determined from the one-loop amplitudes because they cannot arise from local counterterm Lagrangians. With a complete calculation at next-to-leading order, it would be possible to fit all the amplitudes (as was done in Ref. [13] for the $|\Delta I| = 1/2$ sector), but we feel that this exercise is not instructive given the large number of free parameters available. In this work, we limit ourselves to study the question of whether the lowest-order predictions are subject to large higher-order corrections.

To compare our theoretical results with experiment, we introduce the amplitudes [3]

$$s = \mathcal{A}^{(\mathrm{S})} , \qquad p = -|\boldsymbol{k}|\mathcal{A}^{(\mathrm{P})} , \tag{3}$$

in the rest frame of the decaying baryon. From these amplitudes, we can extract for the S-waves the $|\Delta I| = 3/2$ components

$$S_3^{(\Lambda)} = \tfrac{1}{\sqrt{3}}\left(\sqrt{2}\,s_{\Lambda \to n\pi^0} + s_{\Lambda \to p\pi^-}\right) , \qquad S_3^{(\Xi)} = \tfrac{2}{3}\left(\sqrt{2}\,s_{\Xi^0 \to \Lambda\pi^0} + s_{\Xi^- \to \Lambda\pi^-}\right) ,$$

$$S_3^{(\Sigma)} = -\sqrt{\tfrac{5}{18}}\left(s_{\Sigma^+ \to n\pi^+} - \sqrt{2}\,s_{\Sigma^+ \to p\pi^0} - s_{\Sigma^- \to n\pi^-}\right) , \tag{4}$$

and the $|\Delta I| = 1/2$ components (for Λ and Ξ decays)

$$S_1^{(\Lambda)} = \tfrac{1}{\sqrt{3}}\left(s_{\Lambda \to n\pi^0} - \sqrt{2}\,s_{\Lambda \to p\pi^-}\right) , \qquad S_1^{(\Xi)} = \tfrac{\sqrt{2}}{3}\left(s_{\Xi^0 \to \Lambda\pi^0} - \sqrt{2}\,s_{\Xi^- \to \Lambda\pi^-}\right) , \tag{5}$$

as well as analogous ones for the P-waves. We can then compute from data the ratios collected in Table I, which show the $|\Delta I| = 1/2$ rule for hyperon decays. The experimental values for S_3 and P_3 are listed in the column labeled "Experiment" in Table II.

TABLE I. Experimental values of ratios of $|\Delta I| = 3/2$ to $|\Delta I| = 1/2$ amplitudes.

$S_3^{(\Lambda)}/S_1^{(\Lambda)}$	$S_3^{(\Xi)}/S_1^{(\Xi)}$	$S_3^{(\Sigma)}/s_{\Sigma^- \to n\pi^-}$	$P_3^{(\Lambda)}/P_1^{(\Lambda)}$	$P_3^{(\Xi)}/P_1^{(\Xi)}$	$P_3^{(\Sigma)}/p_{\Sigma^+ \to n\pi^+}$
0.026 ± 0.009	0.042 ± 0.009	-0.055 ± 0.020	0.031 ± 0.037	-0.045 ± 0.047	-0.059 ± 0.024

To begin discussing our theoretical results, we note that our calculation yields no contributions to the S-wave amplitudes $S_3^{(\Lambda)}$ and $S_3^{(\Xi)}$, as shown in Table II. This only indicates that the two amplitudes are predicted to be smaller than $S_3^{(\Sigma)}$ by about a factor of three because there are nonvanishing contributions from operators that occur at the next order, $\mathcal{O}(m_s/\Lambda_{\chi\mathrm{SB}})$, with $\Lambda_{\chi\mathrm{SB}} \sim 1\,\mathrm{GeV}$ being the scale of chiral-symmetry breaking. (An example of such operators is considered in Refs. [7,12].) The experimental values of $S_3^{(\Lambda)}$ and $S_3^{(\Xi)}$ are seen to support this prediction.

The other four amplitudes are predicted to be nonzero. They depend on the two weak parameters β_{27} and δ_{27} of $\mathcal{L}^{\mathrm{w}}$ (as well as on parameters from the strong Lagrangian, which are already determined), with δ_{27} appearing only in loop diagrams. Since we consider only the nonanalytic part of the loop diagrams, and since the errors in the measurements of the P-wave amplitudes are larger than those in the S-wave amplitudes, we can take the point of view that we will extract the value of β_{27} by fitting the tree-level $S_3^{(\Sigma)}$ amplitude to experiment, and then treat the tree-level P-waves as predictions and the loop results as a measure of the uncertainties of the lowest-order predictions.

TABLE II. Summary of results for $|\Delta I| = 3/2$ components of the S- and P-wave amplitudes to $\mathcal{O}(m_s \ln m_s)$. We use the parameter values $\beta_{27} = \delta_{27} = -0.068\sqrt{2}\, f_\pi G_\mathrm{F} m_\pi^2$ and a subtraction scale $\mu = 1\,\mathrm{GeV}$.

Amplitude	Experiment	Theory		
		Tree	Octet	Decuplet
		$\mathcal{O}(1)$	$\mathcal{O}(m_s \ln m_s)$	$\mathcal{O}(m_s \ln m_s)$
$S_3^{(\Lambda)}$	-0.047 ± 0.017	0	0	0
$S_3^{(\Xi)}$	0.088 ± 0.020	0	0	0
$S_3^{(\Sigma)}$	-0.107 ± 0.038	-0.107	-0.089	-0.084
$P_3^{(\Lambda)}$	-0.021 ± 0.025	0.012	0.005	-0.060
$P_3^{(\Xi)}$	0.022 ± 0.023	-0.037	-0.024	0.065
$P_3^{(\Sigma)}$	-0.110 ± 0.045	0.032	0.015	-0.171

Thus, we obtain $\beta_{27} = -0.068\sqrt{2}\, f_\pi G_\mathrm{F} m_\pi^2$, and the resulting P-wave amplitudes are placed in the column labeled "Tree" in Table II. These lowest-order predictions are not impressive, but they have the right order of magnitude and differ from the central value of the measurements by at most three standard deviations. For comparison, in the $|\Delta I| = 1/2$ case the tree-level predictions for the P-wave amplitudes are completely wrong [1,3], differing from the measurements by factors of up to 30.

To address the reliability of the leading-order predictions, we look at our calculation of the one-loop corrections, presented in two columns in Table II. The numbers in the column marked "Octet" come from all loop diagrams that do not have any decuplet-baryon lines, with β_{27} being the only weak parameter in the diagrams. Contributions of loop diagrams with decuplet baryons depend on one additional constant, δ_{27}, which cannot be fixed from experiment as it does not appear in any of the observed weak decays of a decuplet baryon. To illustrate the effect of these terms, we choose $\delta_{27} = \beta_{27}$, a choice consistent with dimensional analysis and the normalization of $\mathcal{L}^\mathrm{w}$, and collect the results in the column labeled "Decuplet".

We can see that some of the loop corrections in Table II are comparable to or even larger than the lowest-order results even though they are expected to be smaller by about a factor of $M_K^2/(4\pi f_\pi)^2 \approx 0.2$. These large corrections occur when several different diagrams yield contributions that add up constructively, resulting in deviations of up to an order of magnitude from the power-counting expectation. This is an inherent flaw in a perturbative calculation where the expansion parameter is not sufficiently small. We can, therefore, say that these numbers are consistent with naive expectations.

Although the one-loop corrections are large, they are all much smaller than their counterparts in $|\Delta I| = 1/2$ transitions, where they can be as large as 15 times the lowest-order amplitude in the case of the P-wave in $\Sigma^+ \to n\pi^+$. In that case, the discrepancy was due to an anomalously small lowest-order prediction arising from the cancellation of two nearly identical terms [3].

In conclusion, we have presented a discussion of $|\Delta I| = 3/2$ amplitudes for hyperon nonleptonic decays in χPT. At leading order these amplitudes are described in terms of only one weak parameter. This parameter can be fixed from the observed value of the S-wave amplitudes in Σ decays. After fitting this number, we have predicted the P-waves and used our one-loop calculation to discuss uncertainties of the lowest-order predictions. Our predictions are not contradicted by current data, but current experimental errors are too large for a meaningful conclusion. We have shown that the one-loop nonanalytic corrections have the relative size expected from naive power counting. The combined efforts of E871 and KTeV experiments at Fermilab could give us improved accuracy in the measurements of some of the decay modes that we have discussed and allow a more quantitative comparison of theory and experiment.

III. $|\Delta I| = 3/2$ DECAYS OF THE Ω^-

In the heavy-baryon formalism, we can write the amplitude for $\Omega^- \to \Xi\pi$ as [9]

$$i\mathcal{M}_{\Omega^- \to \Xi\pi} \; = \; G_{\mathrm{F}} m_\pi^2 \; \bar{u}_\Xi \, \mathcal{A}^{(\mathrm{P})}_{\Omega^-\Xi\pi} \, k_\mu \, u_\Omega^\mu \; \equiv \; G_{\mathrm{F}} m_\pi^2 \; \bar{u}_\Xi \, \frac{\alpha^{(\mathrm{P})}_{\Omega^-\Xi}}{\sqrt{2} \, f_\pi} \, k_\mu \, u_\Omega^\mu \; , \tag{6}$$

where the u's are baryon spinors, k is the outgoing four-momentum of the pion, and only the dominant P-wave piece of the amplitude is included. We will consider only the P-wave because, experimentally, the asymmetry parameter in these decays is small and consistent with zero [8], indicating that they are dominated by a P-wave.

From the measured decay rates, we obtain [9]

$$\mathcal{A}^{(\mathrm{P})}_{\Omega^-\Xi^-\pi^0} \; = \; (3.31 \pm 0.08) \, \mathrm{GeV}^{-1} \; , \qquad \mathcal{A}^{(\mathrm{P})}_{\Omega^-\Xi^0\pi^-} \; = \; (5.48 \pm 0.09) \, \mathrm{GeV}^{-1} \; . \tag{7}$$

Upon defining the $|\Delta I| = 1/2, \, 3/2$ amplitudes

$$\alpha_1^{(\Omega)} \; \equiv \; \frac{1}{\sqrt{3}} \left(\alpha^{(\mathrm{P})}_{\Omega^-\Xi^-} + \sqrt{2} \, \alpha^{(\mathrm{P})}_{\Omega^-\Xi^0} \right) \; , \qquad \alpha_3^{(\Omega)} \; \equiv \; \frac{1}{\sqrt{3}} \left(\sqrt{2} \, \alpha^{(\mathrm{P})}_{\Omega^-\Xi^-} - \alpha^{(\mathrm{P})}_{\Omega^-\Xi^0} \right) \; , \tag{8}$$

respectively, we can extract the ratio

$$\alpha_3^{(\Omega)} / \alpha_1^{(\Omega)} \; = \; -0.072 \pm 0.013 \; , \tag{9}$$

which is higher than the corresponding ratios in octet-hyperon decays listed in Table I, but not significantly so.

Although the size of this ratio is not clear evidence for violation of the $|\Delta I| = 1/2$ rule in Ω^- decays, it leads to a different question, that of the compatibility of the measurements of these decays and those of the octet-hyperon decays. To address this question, we will first extract a $|\Delta I| = 3/2$ coupling from $\Omega^- \to \Xi\pi$ decays and then examine its contribution to the octet-hyperon decays.

Employing standard group-theory techniques, we find two different operators that transform as $(27_{\mathrm{L}}, 1_{\mathrm{R}})$ and generate $\Delta S = 1$, $|\Delta I| = 3/2$ transitions involving Ω^- fields. We write them as

$$\mathcal{L}_1^{\mathrm{w}} \; = \; T_{ij,kl} \, \xi_{ka} \xi_{lb} \left(\mathcal{C}_{27} \, I_{ab,cd} + \mathcal{C}_{27}' \, I'_{ab,cd} \right) \xi^\dagger_{ci} \xi^\dagger_{dj} \; , \tag{10}$$

where $\mathcal{C}_{27}$ and $\mathcal{C}_{27}'$ are the weak parameters for the two operators, the baryon fields are contained in the tensors I and I', and additional details can be found in Ref. [9]. This Lagrangian contains the terms

$$\mathcal{L}^{\mathrm{w}}_{\Omega^- B\phi} = \frac{\mathcal{C}_{27}}{f} 6 \left(-\sqrt{2} \, \bar{\Sigma}_v^- \, \partial^\mu K^0 + 2 \, \bar{\Sigma}_v^0 \, \partial^\mu K^+ - 2 \, \bar{\Xi}_v^- \, \partial^\mu \pi^0 + \sqrt{2} \, \bar{\Xi}_v^0 \, \partial^\mu \pi^+ \right) \Omega^-_{v\mu}$$

$$+ \frac{\mathcal{C}_{27}'}{f} 2 \left(\sqrt{2} \, \bar{\Sigma}_v^- \, \partial^\mu K^0 - 2 \, \bar{\Sigma}_v^0 \, \partial^\mu K^+ - 2 \, \bar{\Xi}_v^- \, \partial^\mu \pi^0 + \sqrt{2} \, \bar{\Xi}_v^0 \, \partial^\mu \pi^+ \right) \Omega^-_{v\mu} \; . \tag{11}$$

From this expression, one can see that the decay modes $\Omega^- \to \Xi\pi$ measure the combination $3\mathcal{C}_{27} + \mathcal{C}_{27}'$. Since the decays $\Omega^- \to \Sigma K$ are kinematically forbidden, and since three body decays of the Ω^- are poorly measured, it is not possible at present to extract these two constants separately.

At tree level, the P-wave amplitudes arise from contact diagrams generated by $\mathcal{L}_1^{\mathrm{w}}$ in (10) and are given by

$$\alpha^{(\mathrm{P})}_{\Omega^-\Xi^-} \; = \; -4\sqrt{2} \, (3\mathcal{C}_{27} + \mathcal{C}_{27}') \; , \qquad \alpha^{(\mathrm{P})}_{\Omega^-\Xi^0} \; = \; 4 \, (3\mathcal{C}_{27} + \mathcal{C}_{27}') \; . \tag{12}$$

The value of the constant $3\mathcal{C}_{27} + \mathcal{C}_{27}'$ is then found to be

$$3\mathcal{C}_{27} + \mathcal{C}_{27}' \; = \; (8.7 \pm 1.6) \times 10^{-3} \, G_{\mathrm{F}} m_\pi^2 \; . \tag{13}$$

This value is consistent with power counting, being suppressed by approximately a factor of $\Lambda_{\chi\mathrm{SB}}$ with respect to the parameter β_{27} previously discussed.

We now address the question of the size of the contribution of $\mathcal{L}_1^{\mathrm{w}}$ in (10) to the $|\Delta I| = 3/2$ decays of octet hyperons at one-loop. We again keep only the nonanalytic terms of the loop results. As an illustration of the effect of these terms on the octet-hyperon decays, we present numerical results in Table III, where we look at four simple scenarios to satisfy Eq. (13) in terms of only one parameter. Interestingly, there are no contributions to $S_3^{(\Lambda)}$ and $S_3^{(\Xi)}$ as before, and so only the amplitudes predicted to be nonzero are displayed. For comparison, we show in the same Table the experimental value of the amplitudes as well as the best theoretical fit at $\mathcal{O}(m_s \log m_s)$ obtained in Ref. [7].

TABLE III. New $|\Delta I| = 3/2$ contributions to S- and P-wave hyperon decay amplitudes compared with experiment and with the best theoretical fit of Ref. [7]. Here $\mathcal{C}_{27}$ and $\mathcal{C}'_{27}$ are given in units of $10^{-3}\,G_{\mathrm{F}}m_\pi^2$, and their values are chosen to fit the $\Omega^- \to \Xi\pi$ decays.

Amplitude	Experiment	Theory Ref. [7]	Theory, new contributions with $3\mathcal{C}_{27} + \mathcal{C}'_{27} = 8.7$			
			$\mathcal{C}'_{27} = 0$	$\mathcal{C}_{27} = 0$	$\mathcal{C}'_{27} = \mathcal{C}_{27}$	$\mathcal{C}'_{27} = -\mathcal{C}_{27}$
$S_3^{(\Sigma)}$	-0.107 ± 0.038	-0.120	-0.29	0.52	-0.09	-0.70
$P_3^{(\Lambda)}$	-0.021 ± 0.025	-0.023	0.02	-0.04	0.01	0.05
$P_3^{(\Xi)}$	0.022 ± 0.023	0.027	-0.10	0.10	-0.05	-0.20
$P_3^{(\Sigma)}$	-0.110 ± 0.045	-0.066	0.05	-0.09	0.02	0.13

The new terms calculated here (with $\mu = 1$ GeV), induced by $\mathcal{L}_1^{\mathrm{w}}$, are of higher order in m_s and are therefore expected to be at most comparable to the best theoretical fit. A quick glance at Table III shows that in some cases the new contributions are much larger. Another way to gauge the size of the new contributions is to compare them with the experimental error in the octet-hyperon decay amplitudes. Since the theory provides a good fit at $\mathcal{O}(m_s \log m_s)$ [7], we would like the new contributions (which are of higher order in m_s) to be at most at the level of the experimental error. From Table III, we see that in some cases the new contributions are significantly larger than these errors. In a few cases they are significantly larger than the experimental amplitudes. All this indicates to us that the measured $\Omega^- \to \Xi\pi$ decay rates imply a $|\Delta I| = 3/2$ amplitude that may be too large and in contradiction with the $|\Delta I| = 3/2$ amplitudes measured in octet-hyperon decays.

However, it is premature to conclude that the measured rates of the $\Omega^- \to \Xi\pi$ decays are incorrect because none of the contributions to octet-hyperon decay amplitudes is proportional to the same combination of parameters measured in $\Omega^- \to \Xi\pi$ decays, $3\mathcal{C}_{27} + \mathcal{C}'_{27}$. It is possible to construct linear combinations of the four amplitudes $S_3^{(\Sigma)}$, $P_3^{(\Sigma)}$, $P_3^{(\Lambda)}$ and $P_3^{(\Xi)}$ that are proportional to $3\mathcal{C}_{27} + \mathcal{C}'_{27}$. We find that the most sensitive one is

$$\left(S_3^{(\Sigma)} - 4.2 P_3^{(\Xi)}\right)_{\mathrm{Exp}} = -0.2 \pm 0.1\,, \tag{14}$$

where we have simply combined the errors in quadrature. The contribution from $\mathcal{L}_1^{\mathrm{w}}$ to this combination is

$$\left(S_3^{(\Sigma)} - 4.2 P_3^{(\Xi)}\right)_{\mathrm{Theory, new}} \approx 13\left(3\mathcal{C}_{27} + \mathcal{C}'_{27}\right) \approx 0.1\,, \tag{15}$$

which falls within the error in the measurement.

Our conclusion is that the current measured rates for $\Omega^- \to \Xi\pi$ implies a $|\Delta I| = 3/2$ amplitude that appears large enough to be in conflict with measurements of $|\Delta I| = 3/2$ amplitudes in octet-hyperon decays. Nevertheless, within current errors and without any assumptions about the relative size of $\mathcal{C}_{27}$ and $\mathcal{C}'_{27}$, the two sets of measurements are not in conflict.

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[1] J. Bijnens, H. Sonoda and M. B. Wise, Nucl. Phys. B **261**, 185 (1985).

[2] H. Georgi, *Weak Interactions and Modern Particle Theory* (The Benjamin/Cummings Publishing Company, Menlo Park, 1984); J. F. Donoghue, E. Golowich and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, 1992).

[3] E. Jenkins, Nucl. Phys. B **375**, 561 (1992).

[4] E. Jenkins and A. Manohar, in *Effective Field Theories of the Standard Model*, edited by U.-G. Meissner (World Scientific, Singapore, 1992).

[5] C. Carone and H. Georgi, Nucl. Phys. B **375**, 243 (1992).

[6] R. P. Springer, hep-ph/9508324.

[7] A. A. El-Hady, J. Tandean and G. Valencia, hep-ph/9808322, to appear in Nucl. Phys. A.

[8] Review of Particle Physics. C. Caso et. al., Eur. Phys. J. C **3**, 1 (1998).

[9] J. Tandean and G. Valencia, Phys. Lett. B **452**, 395 (1999).

[10] E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991).

[11] E. Jenkins and A. Manohar, Phys. Lett. B **259**, 353 (1991).

[12] X.-G. He and G. Valencia, Phys. Lett. B **409**, 469 (1997); Erratum, Phys. Lett. B **418**, 443 (1998).

[13] B. Borasoy and B. R. Holstein, Eur. Phys. J. C **6**, 85 (1999).