Glueball Mass Spectrum from Supergravity

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We review the calculation of the spectrum of glueball masses in non-supersymmetric Yang-Mills theory using the conjectured duality between supergravity and large N gauge theories. The glueball masses are obtained by solving the supergravity wave equations in a black hole geometry. The glueball masses found this way are in unexpected agreement with the available lattice data. We also show how to use a modified version of the duality based on rotating branes to calculate the glueball mass spectrum with some of the Kaluza-Klein states of the supergravity theory decoupled from the spectrum.

I. INTRODUCTION

Maldacena's conjecture [1] relates $\mathcal{N}=4$ supersymmetric $SU(N)$ gauge theories in the large N limit to Type IIB string theory on an $\mathrm{AdS}_5 \times \mathbf{S}^5$ background, where AdS_5 is a five dimensional anti-de Sitter space. The metric of this space is given by

$$\frac{ds^2}{l_s^2\sqrt{4\pi g_s N}} = \rho^{-2}d\rho^2 + \rho^2 \sum_{i=1}^{4} dx_i^2 + d\Omega_5^2 \tag{1}$$

where l_s is the string length related to the superstring tension, g_s is the string coupling constant and $d\Omega_5$ is the line element on $\mathbf{S}^5$. The $x_{1,2,3,4}$ directions in AdS_5 correspond to $\mathbf{R}^4$ where the gauge theory lives. The gauge coupling constant g_4 of the 4D theory is related to the string coupling constant g_s by $g_4^2 = g_s$. In the 't Hooft limit ($N \to \infty$ with $g_4^2 N = g_s N$ fixed), the string coupling constant vanishes $g_s \to 0$. Therefore we can study the 4D theory using the first quantized string theory in the AdS space (1). Moreover if $g_s N \gg 1$, the curvature of the AdS space is small and the string theory is approximated by classical supergravity. Witten extended this proposal to non-supersymmetric theories [2]. In his setup supersymmetry is broken by heating up the $\mathcal{N}=4$ theory, which corresponds to putting the four dimensional theory on a circle and assigning anti-periodic boundary conditions to the fermions. In this case the fermions will get a supersymmetry breaking mass term of the order $T = 1/2\pi R$, where R is the radius of the compact coordinate and T is the corresponding temperature, while the scalars (not protected by supersymmetry anymore) will get masses from loop corrections. Thus in the $T \to \infty$ limit this should reproduce a pure (3 dimensional) $SU(N)$ theory in the large N limit, which we will refer to as QCD_3. On the string theory side this corresponds to replacing the anti-de Sitter metric by a Schwarzschild metric describing a black hole in the anti-de Sitter space. This metric is given by

$$\frac{ds^2}{l_s^2\sqrt{4\pi g_s N}} = \left(\rho^2 - \frac{b^4}{\rho^2}\right)^{-1} d\rho^2 + \left(\rho^2 - \frac{b^4}{\rho^2}\right) d\tau^2 + \rho^2 \sum_{i=1}^{3} dx_i^2 + d\Omega_5^2, \tag{2}$$

where τ parameterizes the compactifying circle and the $x^{1,2,3}$ direction corresponding to the $\mathbf{R}^3$ where QCD_3 lives. The horizon of this geometry is located at $\rho = b$ with

$$b = \frac{1}{2R} = \pi T. \tag{3}$$

The supergravity approximation is valid for this theory when the curvature of the space is small, thus when $g_s N \to \infty$. However, in order to obtain the pure gauge theory we have to take the temperature to infinity. In order to keep the intrinsic scale $g_3^2 N = g_4^2 N/R$ of the resulting theory at the scale of QCD, we simultaneously would need to take $g_4^2 N = g_s N \to 0$. Here g_3 is the dimensionful gauge coupling of QCD_3. This is exactly the opposite limit in which the supergravity approximation is applicable! Thus as expected for any strong-weak duality, the weakly coupled classical supergravity theory and the QCD_3 theory are valid in different limits of the 't Hooft coupling $g_4^2 N$.

From the point of view of QCD_3, the radius R of the compactifying circle provides the ultraviolet cutoff scale. Therefore, with the currently available techniques, the Maldacena-Witten conjecture can only be used to study large N QCD with a fixed ultraviolet cutoff R^{-1} in the strong ultraviolet coupling regime, and hope that the results one obtains this way are not very sensitive to removing the cutoff, that is on going from one limit to the other. Since the theory is non-supersymmetric, there is a priori no reason to believe that these two limits have anything to do with each other, since for example there might very well be a phase transition when the 't Hooft coupling is decreased from the very large values where the supergravity description is valid to the small values where the theory should describe QCD_3. Nevertheless, Witten showed that the supergravity theory correctly reproduces several of the qualitative features of a confining 3 dimensional pure gauge theory correctly [2]. In particular, he showed that there is an area law in the Wilson loop and that there is a mass gap in the spectrum, both of which are expected features of a confining gauge theory. Here we will address the question of whether any of the quantitative features of the gauge theories are reproduced as well. In particular, we will calculate the glueball mass spectrum of the theory, and find, that it is in reasonable agreement with recent lattice simulations [3].

II. THE GLUEBALL SPECTRUM IN 3 DIMENSIONS

In this section we will show how to calculate the glueball spectrum of some of the glueballs in the supergravity approximation in the 3 dimensional case. In the following we will use the notation J^{PC} for the glueballs, where J is the glueball spin, and P, C refer to the parity and charge conjugation quantum numbers respectively. In the field theory, one can find operators that have the quantum numbers corresponding to the given glueball states. For example, an operator with quantum numbers 0^{++} is given by $\mathcal{O}_4 = \mathrm{Tr} F^2$, or an operator with quantum numbers 0^{--} is given by $\mathcal{O}_6 = d^{abc} F^a_{\mu\alpha} F^{b\alpha\beta} F^c_{\beta\nu}$. According to the refinement of the Maldacena conjecture given in [4], one should find a supergravity state corresponding to the chiral primary operators of the original $\mathcal{N}=4$ conformal theory, which will couple to the supergravity states on the boundary of the AdS space. Assuming this coupling is maintained while heating the system, we can find the supergravity operators coupling to $\mathcal{O}_4$ and $\mathcal{O}_6$. The dilaton and the R-R scalar of the supergravity theory combine into a complex massless scalar field. Its real and imaginary parts couple to the dimension 4 scalar operators $\mathcal{O}_4 = \mathrm{tr}\ F^2$ and $\tilde{\mathcal{O}}_4 = \mathrm{tr}\ F \wedge F$. The NS-NS and R-R two-forms combine into a complex-valued antisymmetric field $A_{\mu\nu}$, polarized along the $\mathbf{R}^4$. Its $(\mathrm{AdS\ mass})^2 = 16$ and thus one can show that it couples to a dimension 6 two-form operator of the $\mathcal{N}=4$ theory. This operator has been identified as the operator $\mathcal{O}_6$ [5,6]. With this knowledge we would like to calculate the actual glueball mass spectrum corresponding to these operators $\mathcal{O}_4$ and $\mathcal{O}_6$. In field theory, in order to calculate the masses of these states one would need to evaluate the correlators $\langle \mathcal{O}_4(x)\mathcal{O}_4(y)\rangle = \sum_i c_i e^{-m_i|x-y|}$, where the m_i's are the glueball masses. According to the refinement of the Maldacena conjecture [4], this just amounts to solving the supergravity wave equations for the fields that couple to these operators on the boundary. In the case of the 0^{++} glueballs, we need to find the solutions of the dilaton equations of motion of the form $\Phi = f(\rho)e^{ikx}$. This is because in the supergravity theory on $\mathrm{AdS}_5 \times \mathbf{S}^5$, the Kaluza-Klein modes on the $\mathbf{S}^5$ can be classified according to the spherical harmonics of the $\mathbf{S}^5$, which form representations of the isometry group $SO(6)$ (which is the R-symmetry group of the $\mathcal{N}=4$ theory). When we put the theory at finite temperature, the states carrying non-trivial $SO(6)$ quantum numbers should eventually decouple from the spectrum, thus the glueballs should be identified with the $SO(6)$ singlet states, which implies a solution of the form $\Phi = f(\rho)e^{ikx}$ for the dilaton as mentioned above. Thus we will look for normalizable regular solutions to the dilaton equation of motion which will give a discrete spectrum with the glueball masses determined as $k_i^2 = -M_i^2$.

In the supergravity description we have to solve the classical equation of motion of the massless dilaton,

$$\partial_\mu \left[\sqrt{g}\partial_\nu \Phi g^{\mu\nu}\right] = 0 \ , \tag{4}$$

on the AdS_5 black hole background (2). Plugging the ansatz $\Phi = f(\rho)e^{ikx}$ into this equation and using the metric of (2) one obtains the following differential equation for f:

$$\rho^{-1}\frac{d}{d\rho}\left(\left(\rho^4 - b^4\right)\rho\frac{df}{d\rho}\right) - k^2 f = 0 \tag{5}$$

Since the glueball mass M^2 is equal to $-k^2$, the task is to solve this equation as an eigenvalue problem for k^2. In the following we set $b = 1$, so the masses are computed in units of b. We need to find normalizable solutions to this equations which are also regular at the horizon. For large ρ, the black hole metric (2) asymptotically approaches the AdS metric, and the behavior of the solution for a p-form for large ρ takes the form ρ^λ, where λ is determined from the mass m of the supergravity field:

$$m^2 = \lambda(\lambda + 4 - 2p)\ . \tag{6}$$

Indeed both (5) and (6) give the asymptotic forms $f \sim 1, \rho^{-4}$, and only the later is a normalizable solution [2]. Changing variables to $f = \psi/\rho^4$ we have:

$$\left(\rho^2 - \rho^6\right)\psi'' + \left(3\,\rho^5 - 7\rho\right)\psi' + \left(16 + k^2\rho^2\right)\psi = 0 \tag{7}$$

For large ρ this equation can be solved by series solution with negative even powers:

$$\psi = \Sigma_{n=0}^{\infty} a_{2n}\rho^{-2n} \tag{8}$$

Since the normalization is arbitrary we can set $a_0 = 1$. The first few coefficients are given by:

$$a_2 = \frac{k^2}{12},\quad a_4 = \frac{1}{2} + \frac{k^4}{384},\quad a_6 = \frac{7k^2}{120} + \frac{k^6}{23040}. \tag{9}$$

For $n \geq 5$ the coefficients are given by the recursive relation:

$$(n^2 + 4n)a_n = k^2 a_{n-2} + n^2 a_{n-4}\ . \tag{10}$$

Since the black hole geometry is regular at the horizon $\rho = 1$, k^2 has to be adjusted so that f is also regular at $\rho = 1$ [2]. This can be done numerically in a simple fashion using a "shooting" technique as follows. For a given value of k^2 the equation is numerically integrated from some sufficiently large value of ρ ($\rho \gg k^2$) by matching $f(\rho)$ with the asymptotic solution set by (8) and (9). The glueball mass M is related to the eigenvalues of k^2 by $M^2 = -k^2$ in units of b^2. The results obtained this way, together with the results of the lattice simulations [7] are displayed in Table I. Since the lattice results are in units of string tension, we normalize the supergravity results so that the lightest 0^{++} state agrees with the lattice result. One should also expect a systematic error in addition to the statistical error denoted in Table I for the lattice computations. Similar numerical results have been obtained in [8], while a WKB approximation for the eigenvalues of (5) has been obtained in [9].

TABLE I. 0^{++} glueball masses in QCD_3 coupled to tr $F_{\mu\nu}F^{\mu\nu}$. The lattice results are in units of the square root of the string tension. The denoted error in the lattice results is only the statistical one.

state	lattice, $N = 3$	lattice, $N \to \infty$	supergravity
0^{++}	4.329 ± 0.041	4.065 ± 0.055	4.07 (input)
0^{++*}	6.52 ± 0.09	6.18 ± 0.13	7.02
0^{++**}	8.23 ± 0.17	7.99 ± 0.22	9.92
0^{++***}	-	-	12.80
0^{++****}	-	-	15.67
$0^{++*****}$	-	-	18.54

The 0^{--} glueballs can be dealt with similarly by considering the two-form of the supergravity theory, which couples to the operator $\mathcal{O}_6$. The supergravity equation of motion for the s-wave component of this field is given by

$$\frac{3}{\sqrt{g}}\partial_\mu\left[\sqrt{g}\,\partial_{[\mu'}A_{\mu_1'\mu_2']}\,g^{\mu'\mu}g^{\mu_1'\mu_1}g^{\mu_2'\mu_2}\right]-16g^{\mu_1'\mu_1}g^{\mu_2'\mu_2}A_{\mu_1'\mu_2'}=0, \tag{11}$$

where [ ] denotes antisymmetrization with strength one. For the pseudoscalar component of A_{ij} the equation reduces to

$$\rho\left(\rho^4-1\right)h''+\left(3+\rho^4\right)h'-\left(k^2\,\rho+16\,\rho^3\right)h=0\ , \tag{12}$$

in units where $b=1$. This can be solved similarly as for the case of the 0^{++} glueballs, and the results are displayed in Table II. Since the supergravity method and the lattice gauge theory compute the glueball masses in different units, one cannot compare the absolute values of the lowest glueball mass obtained using these methods. However it makes sense to compare the lowest glueball masses of different quantum numbers. Using Tables I and II, we find that the supergravity results are in good agreement with the lattice gauge theory computation [7]:

$$\begin{aligned}\left(\frac{M_{0^{--}}}{M_{0^{++}}}\right)_{\text{supergravity}}&=1.50\\ \left(\frac{M_{0^{--}}}{M_{0^{++}}}\right)_{\text{lattice}}&=1.45\pm0.08\end{aligned} \tag{13}$$

TABLE II. 0^{--} glueball masses in QCD_3 coupled to $\mathcal{O}_6$. The lattice results are in units of square root of the string tension. The normalization of the supergravity results is the same as in Table I.

state	lattice, $N=3$	lattice, $N\to\infty$	supergravity
0^{--}	6.48 ± 0.09	5.91 ± 0.25	6.10
0^{--*}	8.15 ± 0.16	7.63 ± 0.37	9.34
0^{--**}	9.81 ± 0.26	8.96 ± 0.65	12.37
0^{--***}	-	-	15.33
0^{--****}	-	-	18.26
$0^{--*****}$	-	-	21.16

One can see, that the glueball mass ratios obtained from the supergravity calculation are in reasonable agreement with the lattice results, even though as explained in the introduction these two calculations are in the opposite limits for the 't Hooft coupling. Therefore, it is important to see, how the ratios are modified once corrections due to string theory are taken into account. The leading string theory corrections can be calculated by using the results of [10], who calculated the first α' corrections to the AdS black-hole metric (2). The details of the calculation can be found in [3], here we just give the results for the 0^{++} state:

$$\begin{aligned}M^2_{0^{++}}&=11.59\times(1-2.78\zeta(3)\alpha'^3)\Lambda^2_{UV}\\ M^2_{0^{++*}}&=34.53\times(1-2.43\zeta(3)\alpha'^3)\Lambda^2_{UV}\\ M^2_{0^{++**}}&=68.98\times(1-2.28\zeta(3)\alpha'^3)\Lambda^2_{UV}\\ M^2_{0^{++***}}&=114.9\times(1-2.23\zeta(3)\alpha'^3)\Lambda^2_{UV}\\ M^2_{0^{++****}}&=172.3\times(1-2.21\zeta(3)\alpha'^3)\Lambda^2_{UV}\\ M^2_{0^{++*****}}&=241.2\times(1-2.20\zeta(3)\alpha'^3)\Lambda^2_{UV}\ ,\end{aligned} \tag{14}$$

where $\Lambda_{UV}=\frac{1}{2R}$ and the correction to the horizon is given by $b=(1-\frac{15}{8}\zeta(3)\alpha'^3)\frac{1}{2R}$. One can see that the string theory corrections are somewhat uniform for the different excited states of the 0^{++} glueball, and therefore one could hope that these corrections to the ratios of the glueball masses are small. However, it can be seen that this is probably a too optimistic assumption, by considering the Kaluza-Klein partners of the glueball states. As explained above, the glueball states do not carry quantum numbers under the $SO(6)$ isometry, and are also singlets under the $U(1)$ symmetry corresponding to the compact direction τ. The Kaluza-Klein modes however do carry quantum numbers under $SO(6)\times U(1)$, and they do not correspond to any state in the QCD theory, but rather they should decouple in the $R\to0, g_4^2N\to0$ limit from the spectrum. However, in the supergravity limit of finite R, $g_4^2N\to\infty$ these states have masses comparable to the light glueballs [11]. This is simply a consequence of the fact, that the masses of the fermions and scalars carrying the $SO(6)\times U(1)$ quantum numbers is of the order of the temperature T, thus their bound states are expected to also have masses of the order of the temperature. However, since the temperature is the only scale in the theory, and so this will also be the cutoff scale of the QCD theory, and thus the mass scale for the glueballs. In particular, the masses of the KK modes of the 0^{++} glueballs obtained from the dilaton equation by using the ansatz $\Phi = f(\rho)e^{ikx}Y_l(\Omega_5)$ are given by [11]

l	0	1	2	3
M_l^2	11.59	19.43	29.26	41.10
${M_{l^*}}^2$	34.53	48.07	63.60	81.11
${M_{l^{**}}}^2$	68.98	88.24	109.5	132.7

where we have displayed the unnormalized values of the masses of the different KK modes.

One can explicitly see, that the masses of these KK modes are as expected of the same order as the masses of the glueball states. One might hope that even though the supergravity approximation of these masses is of the same order as for the glueballs, string theory corrections will increase the masses of these states compared to the glueball states. Unfortunately, at least the leading string theory corrections calculated in [11,3] do not support this conclusion. The corrections to the first few KK modes are

$$
\begin{aligned}
M_0^2 &= 11.59\times(1-2.78\zeta(3)\alpha'^3)\Lambda_{UV}^2 \\
M_1^2 &= 19.43\times(1-2.73\zeta(3)\alpha'^3)\Lambda_{UV}^2 \\
M_2^2 &= 29.26\times(1-2.74\zeta(3)\alpha'^3)\Lambda_{UV}^2
\end{aligned}
\tag{15}
$$

Thus one can see that the masses of these KK modes in fact do need large α' corrections to remove them from the spectrum of states. Then it is not clear why one would get large corrections to the masses of the KK modes but not to the masses of the glueball states. This situation is clearly unsatisfactory, therefore one may try to improve on it by introducing a different supergravity background, where some of these KK modes are automatically decoupled. We will consider this possibility in the next section where we discuss the construction based on rotating branes [12–14].

III. THE GLUEBALL SPECTRUM IN 4 DIMENSIONS AND THE CONSTRUCTION BASED ON ROTATING BRANES

Results similar to the the ones presented in the previous section can be obtained for the glueball mass spectrum in QCD$_4$ by starting from a slightly different construction where the M-theory 5-brane is wrapped on two circles [2]. The details of these results can be found in [3,15]. Here we will review only the generalized construction based on the rotating M5 brane with one angular momentum, first constructed in [12], and explored in [13]. The metric for this background is given by

$$
\begin{aligned}
ds_{\rm IIA}^2 = \frac{2\pi\lambda A}{3u_0}u\Delta^{1/2}\Big[&4u^2\big(-dx_0^2+dx_1^2+dx_2^2+dx_3^2\big)+\frac{4A^2}{9u_0^2}u^2\,(1-\frac{u_0^6}{u^6\Delta})d\theta_2^2+\frac{4\,du^2}{u^2(1-\frac{a^4}{u^4}-\frac{u_0^6}{u^6})} \\
&+d\theta^2+\frac{\tilde\Delta}{\Delta}\sin^2\theta d\varphi^2+\frac{1}{\Delta}\cos^2\theta d\Omega_2^2-\frac{4a^2Au_0^2}{3u^4\Delta}\sin^2\theta d\theta_2 d\varphi\Big],
\end{aligned}
\tag{16}
$$

where $x_{0,1,2,3}$ are the coordinates along the brane where the gauge theory lives, u is the "radial" coordinate of the AdS space, while the remaining four coordinates parameterize the angular variables of S^4, a is the angular momentum parameter, and we have introduced

$$
\Delta = 1-\frac{a^4\cos^2\theta}{u^4}\,,\qquad \tilde\Delta = 1-\frac{a^4}{u^4}\,,\qquad A\equiv\frac{u_0^4}{u_H^4-\frac{1}{3}a^4}\,,\qquad u_H^6-a^4u_H^2-u_0^6=0\,.
\tag{17}
$$

u_H is the location of the horizon, and the dilaton background and the temperature of the field theory are given by

$$e^{2\phi} = \frac{8\pi}{27} \frac{A^3 \lambda^3 u^3 \Delta^{1/2}}{u_0^3} \frac{1}{N^2} , \qquad R = (2\pi T_H)^{-1} = \frac{A}{3u_0} . \tag{18}$$

Note, that in the limit when $a/u_0 \gg 1$, the radius of compactification R shrinks to zero, thus the KK modes on this compact direction are expected to decouple in this theory when we increase the angular momentum a. In order to find the mass spectrum of the 0^{++} glueballs, we need to again solve the dilaton equations of motion as a function of a. This can be done by plugging the background (16) into the dilaton equation of motion

$$\partial_\mu \left[\sqrt{g} e^{-2\Phi} g^{\mu\nu} \partial_\nu \Phi \right] = 0. \tag{19}$$

For a dilaton ansatz of the form $\Phi = f(u) e^{ikx}$ we obtain the differential equation

$$\partial_u \left[u(u^6 - a^4 u^2 - u_0^6) f'(u) \right] - k^2 u^3 f(u) = 0, \tag{20}$$

which can be solved the same way as explained in the previous section, where the eigenvalues are now a function of the angular momentum parameter a. The results of this are summarized in Table III. Note, that while some of the KK modes decouple in the $a \to \infty$ limit, the 0^{++} glueball mass ratios change only very slightly, showing that the supergravity predictions are robust for these ratios against the change of the angular momentum parameter.

TABLE III. Masses of the first few 0^{++} glueballs in QCD_4, in GeV, from supergravity compared to the available lattice results. The first column gives the lattice result [7,16,17], the second the supergravity result for $a = 0$ while the third the supergravity result in the $a \to \infty$ limit. The change from $a = 0$ to $a = \infty$ in the supergravity predictions is tiny. Note, that for the excited state the supergravity calculation came before the lattice results.

state	lattice, $N = 3$	supergravity $a = 0$	supergravity $a \to \infty$
0^{++}	1.61 ± 0.15	1.61 (input)	1.61 (input)
0^{++*}	2.48 ± 0.18	2.55	2.56
0^{++**}	-	3.46	3.48
0^{++***}	-	4.36	4.40

One can similarly calculate the mass ratios for the 0^{-+} glueballs, by considering the equations of motion of the RR 1-form in the background (16), since on the D4 brane worldvolume this couples to the operator $\mathrm{Tr} F\tilde{F}$. To find the glueball spectrum we have to solve the supergravity equation of motion of the RR 1-form

$$\partial_\nu \left[\sqrt{g} g^{\mu\rho} g^{\nu\sigma} (\partial_\rho A_\sigma - \partial_\sigma A_\rho) \right] = 0 \tag{21}$$

in the background (16). Using the ansatz $A_{\theta_2} = f(u) e^{ikx}$ leads to the differential equation

$$(u^6 - a^4 u^2 - u_0^6) \partial_u \left[u^3 (u^4 - a^4) f'(u) \right] - k^2 u^5 (u^4 - a^4) f(u), \tag{22}$$

which we solve using the same numerical methods as in the previous section. The results are summarized in Table IV. Note, that the change in the 0^{-+} glueball mass is sizeable when going from $a = 0$ to $a \to \infty$, and is in the right direction as suggested by lattice results [16,17].

TABLE IV. Masses of the first few 0^{-+} glueballs in QCD_4, in GeV, from supergravity compared to the available lattice results. The first column gives the lattice result, the second the supergravity result for $a = 0$ while the third the supergravity result in the $a \to \infty$ limit. Note that the change from $a = 0$ to $a = \infty$ in the supergravity predictions is of the order $\sim 25\%$.

state	lattice, $N = 3$	supergravity $a = 0$	supergravity $a \to \infty$
0^{-+}	2.59 $\pm$0.13	2.00	2.56
0^{-+*}	3.64 $\pm$0.18	2.98	3.49
0^{-+**}	-	3.91	4.40
0^{-+***}	-	4.83	5.30

One can also calculate the masses of the different Kaluza-Klein modes in the background of (16). One finds, that as expected from the fact that for $a \to \infty$ the compact circle shrinks to zero, the KK modes on this compact circle decouple from the spectrum, leading to a 4 dimensional field theory in this limit. However, the KK modes of the sphere S^4 do not decouple from the spectrum even in the $a \to \infty$ limit. These conclusions remain unchanged even in the case when one considers the theory with the maximal number of angular momenta (which is two for the case of QCD_4) [14,18].

IV. CONCLUSIONS

We have seen how the Witten extension of Maldacena's conjecture can be used to study pure Yang-Mills theories in the large N limit. These theories reproduce several of the qualitative features of QCD, and one can also study the predictions for the glueball mass spectra. One finds, that the supergravity calculations are in a reasonable agreement with the lattice results, even though they are obtained in the opposite limit of the 't Hooft coupling. It would be very important to understand, whether this unexpected agreement is purely a numerical coincidence or whether there is any deeper reason behind it.

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