KINEMATICS OF AdS_5/CFT_4 DUALITY

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We review the construction of the unitary supermultiplets of the $\mathcal{N}=8$ $d=5$ anti-de Sitter (AdS_5) superalgebra $SU(2,2|4)$, which is the symmetry group of type IIB superstring theory on $AdS_5 \times S^5$, using the oscillator method.

I. INTRODUCTION

Recently a great deal of work has been done on AdS/CFT (anti-de Sitter/conformal field theory) dualities in various dimensions. This activity was primarily started with the original conjecture of Maldacena [1], [2,3] about the relation between the large N limits of certain conformal field theories in d dimensions to M-theory/string theory compactified to $d+1$-dimensional AdS spacetimes. The prime example of AdS/CFT duality is the duality between the large N limit of $\mathcal{N}=4$ $SU(N)$ super Yang-Mills theory in $d=4$ and type IIB superstring theory on $AdS_5 \times S^5$.

In this lecture we will review some results obtained in collaboration with M. Günaydin and M. Zagermann [4], [5] regarding the construction of the unitary supermultiplets of the $\mathcal{N}=8$ $d=5$ anti-de Sitter (AdS_5) superalgebra $SU(2,2|4)$, which is the symmetry group of type IIB superstring theory on $AdS_5 \times S^5$, using the oscillator method. Our results should be relevant for the understanding of the spectrum of type IIB string theory on $AdS_5 \times S^5$ [6].

II. SHORT REVIEW OF THE OSCILLATOR METHOD

In [7] a general oscillator method was developed for constructing the unitary irreducible representations (UIR) of the lowest (or highest) weight type of non-compact groups. The oscillator method yields the UIR's of lowest weight type of a noncompact group over the Fock space of a set of bosonic oscillators. To achieve this one realizes the generators of the noncompact group as bilinears of sets of bosonic oscillators transforming in a finite dimensional representation of its maximal compact subgroup. The minimal realization of these generators requires either one or two sets of bosonic annihilation and creation operators transforming irreducibly under its maximal compact subgroup. These minimal representations are fundamental in that all the other ones can be obtained from the minimal representations by a simple tensoring procedure.

These fundamental representations are nothing but a generalization of the celebrated remarkable representations of the AdS_4 group $SO(3,2)$ discovered by Dirac [8] long time ago, which were later named singletons [9] (indicating the fact that the remarkable representations of Dirac corresponding to the fields living on the boundary of AdS_4 are singular when the Poincare limit is taken). In the language of the oscillator method, these singleton representations require a single set of oscillators transforming in the fundamental representation of the maximal compact subgroup of the covering group $Sp(4,R)$ of $SO(3,2)$ [10], [13] (a fact that meshes nicely with the name singleton). In some cases (as with the AdS_5 group $SU(2,2)$) the fundamental representations require two sets of oscillators, and they were called doubletons in [12], [14]. The general oscillator construction of the lowest (or highest) weight representations of non-compact supergroups (i.e. the case when the even subgroup is non-compact) was given in [11]. The oscillator method was further developed and applied to the spectra of Kaluza-Klein supergravity theories in references [12], [13], [14].

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A non-compact group G that admits unitary representations of the lowest weight type has a maximal compact subgroup G^0 of the form $G^0 = H \times U(1)$ with respect to whose Lie algebra g^0 one has a three grading of the Lie algebra g of G,

$$g = g^{-1} \oplus g^0 \oplus g^{+1} \tag{1}$$

which simply means that the commutators of elements of grade k and l satisfy

$$[g^k, g^l] \subseteq g^{k+l}. \tag{2}$$

Here $g^{k+l} = 0$ for $|k+l| > 1$.

For example, for $SU(1,1)$ this corresponds to the standard decomposition $g = L_+ \oplus L_0 \oplus L_-$ where

$$[L_0, L_\pm] = \pm L_\pm, \quad [L_+, L_-] = 2L_0. \tag{3}$$

The three grading is determined by the generator E of the $U(1)$ factor of the maximal compact subgroup

$$[E, g^{+1}] = g^{+1}, \quad [E, g^{-1}] = -g^{-1}, \quad [E, g^0] = 0. \tag{4}$$

In most physical applications E turns out to be the energy operator. In such cases the unitary lowest weight representations correspond to positive energy representations.

The bosonic annihilation and creation operators in terms of which one realizes the generators of G transform typically in the fundamental and its conjugate representation of H. In the Fock space $\mathcal{H}$ of all the oscillators one chooses a set of states $|\Omega\rangle$ which transform irreducibly under $H \times U(1)$ and are annihilated by all the generators in g^{-1}. Then by acting on $|\Omega\rangle$ with generators in g^{+1} one obtains an infinite set of states

$$|\Omega\rangle, \quad g^{+1}|\Omega\rangle, \quad g^{+1}g^{+1}|\Omega\rangle, \ldots \tag{5}$$

which form an UIR of the lowest weight (positive energy) type of G. The infinite set of states thus obtained corresponds to the decomposition of the UIR of G with respect to its maximal compact subgroup.

As we have already emphasized, whenever we can realize the generators of G in terms of a single set of bosonic creation (and annihilation) operators transforming in an irreducible representation (and its conjugate) of the compact subgroup H then the corresponding UIRs will be called singleton representations and there exist two such representations for a given group G. For the AdS group in $d = 4$ the singleton representations correspond to scalar and spinor fields . In certain cases we need a minimum of two sets of bosonic creation and annihilation operators transforming irreducibly under H to realize the generators of G. In such cases the corresponding UIRs are called doubleton representations and there exist infinitely many doubleton representations of G corresponding to fields of different "spins".

Even though the Poincare limit of the singleton (or doubleton) representations is singular, the tensor product of two singleton (or doubleton) representations decomposes into an infinite set of "massless" irreducible representations which do have a smooth Poincare limit [9], [10], [13]. Furthermore, tensoring more than two singletons or doubletons representations leads to "massive" representations of AdS groups and supergroups.

The relation between Maldacena's conjecture and the dynamics of the singleton and doubleton fields that live on the boundary of AdS spacetimes was reviewed in [15], [16], [17].

III. THE SUPERALGEBRA $SU(2,2|4)$

The centrally extended symmetry supergroup of type IIB superstring theory on $AdS_5 \times S^5$ is the supergroup $SU(2,2|4)$ with the even subgroup $SU(2,2) \times SU(4) \times U(1)_Z$, where $SU(4)$ is the double cover of $SO(6)$, the isometry group of the five sphere [12]. The Abelian $U(1)_Z$ generator, which we will call Z, commutes with all the other generators and acts like a central charge. Therefore, $SU(2,2|4)$ is not a simple Lie superalgebra. By factoring out this Abelian ideal one obtains a simple Lie superalgebra, denoted by $PSU(2,2|4)$, whose even subalgebra is simply $SU(2,2) \times SU(4)$. We consider below the centrally extended supergroup $SU(2,2|4)$ [4]. The representations of $PSU(2,2|4)$ correspond simply to representations of $SU(2,2|4)$ with $Z=0$. We should also note that both $SU(2,2|4)$ and $PSU(2,2|4)$ admit an outer automorphism $U(1)_Y$ that can be identified with the $U(1)$ subgroup of the $SU(1,1)$ symmetry of IIB supergravity in $d=10$ [12]. $SU(2,2|4)$ can be interpreted as the $\mathcal{N}=8$ extended AdS superalgebra in $d=5$ or as the $\mathcal{N}=4$ extended conformal superalgebra in $d=4$.

The algebra of $\mathcal{N}$-extended conformal supersymmetry in $d=4$ can be written in a covariant form as follows ($i,j=1,\ldots,\mathcal{N}$; $a,b=0,1,2,3,5,6$) [18]:

$$
\begin{gathered}
[\Xi_i, M_{ab}] = \Sigma(M_{ab})\Xi_i, [\bar{\Xi}^i, M_{ab}] = -\bar{\Xi}^i\Sigma(M_{ab}) \\
\{\Xi_i, \Xi_j\} = \{\bar{\Xi}^i, \bar{\Xi}^j\} = 0, \{\Xi_i, \bar{\Xi}^j\} = 2\delta_i^j \Sigma(M^{ab})M_{ab} - 4B_i^j \\
[B_i^j, M_{ab}] = 0, [B_i^j, B_k^l] = \delta_i^l B_k^j - \delta_k^j B_i^l \\
[\Xi_i, B_j^k] = \delta_i^k \Xi_j - \frac{1}{4}\delta_j^k \Xi_i, [\bar{\Xi}^i, B_j^k] = -\delta_j^i \bar{\Xi}^k + \frac{1}{4}\delta_j^k \bar{\Xi}^i,
\end{gathered}
\tag{6}
$$

where the (four component) conformal spinor Ξ is defined in terms of the the chiral components of the Lorentz spinors Q and S (the generators of Poincaré and S type supersymmetry) as

$$
\Xi \equiv \begin{pmatrix} Q_\alpha \\ \bar{S}^{\dot{\alpha}} \end{pmatrix}.
\tag{7}
$$

The B_i^j are the generators of the internal (R-)symmetry group $U(\mathcal{N})$ and the $\Sigma(M_{ab})$ are 4×4 matrices generating an irreducible representation of $SU(2,2)$ [4], [18].

The superalgebra $SU(2,2|4)$ has a three graded decomposition with respect to its compact subsuperalgebra $SU(2|2)\times SU(2|2)\times U(1)$

$$
g = L^+ \oplus L^0 \oplus L^-,
\tag{8}
$$

where

$$
[L^0, L^\pm] \subseteq L^\pm, \quad [L^+, L^-] \subseteq L^0, \quad [L^+, L^+] = 0 = [L^-, L^-].
\tag{9}
$$

Here L^0 represents the generators of $SU(2|2)\times SU(2|2)\times U(1)$.

The Lie superalgebra $SU(2,2|4)$ can be realized in terms of bilinear combinations of bosonic and fermionic annihilation and creation operators ξ_A ($\xi^A = \xi_A{}^\dagger$) and η_M ($\eta^M = \eta_M{}^\dagger$) which transform covariantly and contravariantly under the two $SU(2|2)$ subsupergroups of $SU(2,2|4)$ [4,10–12]

$$
\xi_A = \begin{pmatrix} a_i \\ \alpha_\gamma \end{pmatrix}, \quad \xi^A = \begin{pmatrix} a^i \\ \alpha^\gamma \end{pmatrix}
\tag{10}
$$

and

$$
\eta_M = \begin{pmatrix} b_r \\ \beta_x \end{pmatrix}, \quad \eta^M = \begin{pmatrix} b^r \\ \beta^x \end{pmatrix}
\tag{11}
$$

with $i,j=1,2$; $\gamma,\delta=1,2$; $r,s=1,2$; $x,y=1,2$ and

$$
[a_i, a^j] = \delta_i^j, \quad \{\alpha_\gamma, \alpha^\delta\} = \delta_\gamma^\delta
\tag{12}
$$

$$
[b_r, b^s] = \delta_r^s, \quad \{\beta_x, \beta^y\} = \delta_x^y.
\tag{13}
$$

Annihilation and creation operators are labelled by lower and upper indices, respectively. The generators of $SU(2,2|4)$ are given in terms of the above superoscillators schematically as

$$L^- = \vec{\xi}_A \cdot \vec{\eta}_M, \quad L^0 = \vec{\xi}^A \cdot \vec{\xi}_B \oplus \vec{\eta}^M \cdot \vec{\eta}_N, \quad L^+ = \vec{\xi}^A \cdot \vec{\eta}^M, \tag{14}$$

where the arrows over ξ and η again indicate that we are taking an arbitrary number P of "generations" of superoscillators and the dot represents the summation over the internal index $K = 1, ..., P$, i.e $\vec{\xi}_A \cdot \vec{\eta}_M \equiv \sum_{K=1}^{P} \xi_A(K)\eta_M(K)$.

The even subgroup $SU(2,2) \times SU(4) \times U(1)_Z$ is obviously generated by the di-bosonic and di-fermionic generators. In particular, one recovers the $SU(2,2)$ generators in terms of the bosonic oscillators:

$$\begin{aligned} L_i^j &= \vec{a}^j \cdot \vec{a}_i - \frac{1}{2}\delta_i^j N_a, \quad R_s^r = \vec{b}^r \cdot \vec{b}_s - \frac{1}{2}\delta_s^r N_b \\ E &= \frac{1}{2}\{\vec{a}^i \cdot \vec{a}_i + \vec{b}_r \cdot \vec{b}^r\} = \frac{1}{2}\{N_a + N_b + 2P\} \\ L_{ir}^- &= \vec{a}_i \cdot \vec{b}_r, \quad L^{+ri} = \vec{b}^r \cdot \vec{a}^i \end{aligned} \tag{15}$$

satisfying

$$[L_{ir}^-, L^{+sj}] = \delta_r^s L_i^j + \delta_i^j R_r^s + \delta_i^j \delta_r^s E. \tag{16}$$

Here, $N_a \equiv \vec{a}^i \cdot \vec{a}_i$, $N_b \equiv \vec{b}^r \cdot \vec{b}_r$ are the bosonic number operators.

Similarly, the $SU(4)$ generators in their $SU(2) \times SU(2) \times U(1)$ basis are expressed in terms of the fermionic oscillators α and β:

$$\begin{aligned} A_\gamma^\delta &= \vec{\alpha}^\delta \cdot \vec{\alpha}_\gamma - \frac{1}{2}\delta_\gamma^\delta N_\alpha, \quad B_x^y = \vec{\beta}^y \cdot \vec{\beta}_x - \frac{1}{2}\delta_x^y N_\beta \\ C &= \frac{1}{2}\{-\vec{\alpha}^\delta \cdot \vec{\alpha}_\delta + \vec{\beta}_x \cdot \vec{\beta}^x\} = \frac{1}{2}\{-N_\alpha - N_\beta + 2P\} \\ L_{\gamma x}^- &= \vec{\alpha}_\gamma \cdot \vec{\beta}_x, \quad L^{+x\gamma} = \vec{\beta}^x \cdot \vec{\alpha}^\gamma \end{aligned} \tag{17}$$

with the closure relation

$$[L_{\gamma x}^-, L^{+y\delta}] = -\delta_x^y A_\gamma^\delta - \delta_\gamma^\delta B_x^y + \delta_x^y \delta_\gamma^\delta C. \tag{18}$$

Here $N_\alpha = \vec{\alpha}^\delta \cdot \vec{\alpha}_\delta$ and $N_\beta = \vec{\beta}^x \cdot \vec{\beta}_x$ are the fermionic number operators.

Finally, the central charge-like $U(1)_Z$ generator Z is given by

$$Z = \frac{1}{2}\{N_a + N_\alpha - N_b - N_\beta\}. \tag{19}$$

Analogously, the odd generators are given by products of bosonic and fermionic oscillators and satisfy the following closure relations

$$\begin{aligned} \{\vec{a}_i \cdot \vec{\beta}_x, \vec{\beta}^y \cdot \vec{a}^j\} &= \delta_x^y L_i^j - \delta_i^j B_x^y + \frac{1}{2}\delta_x^y \delta_i^j (E + C + Z) \\ \{\vec{\alpha}_\gamma \cdot \vec{b}_r, \vec{b}^s \cdot \vec{\alpha}^\delta\} &= -\delta_r^s A_\gamma^\delta + \delta_\gamma^\delta R_r^s + \frac{1}{2}\delta_r^s \delta_\gamma^\delta (E + C - Z) \\ \{\vec{a}^i \cdot \vec{\alpha}_\gamma, \vec{\alpha}^\delta \cdot \vec{a}_j\} &= \delta_\gamma^\delta L_j^i + \delta_j^i A_\gamma^\delta + \frac{1}{2}\delta_\gamma^\delta \delta_j^i (E - C + Z) \\ \{\vec{b}^r \cdot \vec{\beta}_x, \vec{\beta}^y \cdot \vec{b}_s\} &= \delta_x^y R_s^r + \delta_s^r B_x^y + \frac{1}{2}\delta_x^y \delta_s^r (E - C - Z). \end{aligned} \tag{20}$$

The generator Y of the outer automorphism group $U(1)_Y$ is simply

$$Y = N_\alpha - N_\beta. \tag{21}$$

IV. UNITARY SUPERMULTIPLETS OF $SU(2,2|4)$

To construct a basis for a lowest weight UIR of $SU(2,2|4)$, one starts from a set of states, collectively denoted by $|\Omega\rangle$, in the Fock space of the oscillators a, b, α, β that transforms irreducibly under $SU(2|2) \times SU(2|2) \times U(1)$ and that is annihilated by all the generators $\vec{\xi}_A \cdot \vec{\eta}_M \equiv (\vec{a}_i \cdot \vec{b}_r \oplus \vec{a}_i \cdot \vec{\beta}_x \oplus \vec{\alpha}_\gamma \cdot \vec{b}_r \oplus \vec{\alpha}_\gamma \cdot \vec{\beta}_x)$ of L^-

$$\vec{\xi}_A \cdot \vec{\eta}_M |\Omega\rangle = 0. \tag{22}$$

By acting on $|\Omega\rangle$ repeatedly with L^+, one then generates an infinite set of states that form a UIR of $SU(2,2|4)$

$$|\Omega\rangle, \quad L^{+1}|\Omega\rangle, \quad L^{+1}L^{+1}|\Omega\rangle, ... \tag{23}$$

The irreducibility of the resulting representation of $SU(2,2|4)$ follows from the irreducibility of $|\Omega\rangle$ under $SU(2|2) \times SU(2|2) \times U(1)$. Because of the property (22), $|\Omega\rangle$ as a whole will be referred to as the "lowest weight vector (lwv)" of the corresponding UIR of $SU(2,2|4)$.

In the restriction to the subspace involving purely bosonic oscillators, the above construction reduces to the subalgebra $SU(2,2)$ and its positive energy UIR's. Similarly, when restricted to the subspace involving purely fermionic oscillators, one gets the compact internal symmetry group $SU(4)$ (17), and the above construction yields the representations of $SU(4)$ in its $SU(2) \times SU(2) \times U(1)$ basis. Accordingly, a lowest weight UIR of $SU(2,2|4)$ decomposes into a direct sum of finitely many positive energy UIR's of $SU(2,2)$ transforming in certain representations of the internal symmetry group $SU(4)$. Thus each positive energy UIR of $SU(2,2|4)$ corresponds to a supermultiplet of fields living in AdS_5 or on its boundary.

V. DOUBLETON SUPERMULTIPLETS OF $SU(2,2|4)$

By choosing one pair of super oscillators (ξ and η) in the oscillator realization of $SU(2,2|4)$ (i.e. for $P = 1$), one obtains the so-called doubleton supermultiplets. The doubleton supermultiplets contain only doubleton representations of $SU(2,2)$, i.e. they are multiplets of fields living on the boundary of AdS_5 without a $5d$ Poincaré limit. Equivalently, they can be characterized as multiplets of massless fields in $4d$ Minkowski space that form a UIR of the $\mathcal{N} = 4$ superconformal algebra $SU(2,2|4)$.

The supermultiplet defined by the lwv $|\Omega\rangle = |0\rangle$ of $SU(2,2|4)$ is the unique irreducible CPT self-conjugate doubleton supermultiplet. It is also the supermultiplet of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in $d = 4$ [12].

If we take the following lowest weight vectors

$$|\Omega\rangle = \xi^A|0\rangle \equiv a^i|0\rangle \oplus \alpha^\gamma|0\rangle, \quad |\Omega\rangle = \eta^A|0\rangle \equiv b^r|0\rangle \oplus \beta^x|0\rangle \tag{24}$$

we get a supermultiplet of spin range 3/2 and its CPT conjugate supermultiplet, respectively [4,5].

These two doubleton supermultiplets of spin range 3/2 would occur in the $\mathcal{N} = 4$ super Yang-Mills theory if there is a well-defined conformal (i.e. massless) limit of the 1/4 BPS states described in ref [19]. These 1/4 BPS multiplets are massive representations of the four dimensional $\mathcal{N} = 4$ Poincaré superalgebra with two central charges, one of them saturating the BPS bound. As such, they are equivalent to massive representations of the $\mathcal{N} = 3$ Poincaré superalgebra without central charges. The corresponding multiplet with the lowest spin content (see e.g. [20]) contains 14 scalars, 14 spin 1/2 fermions, six vectors and one spin 3/2 fermion, giving altogether 2^6 states. If a massless limit of such a multiplet exists, it should decompose into two self-conjugate doubleton multiplets plus a doubleton supermultiplet of spin 3/2 plus its CPT conjugate supermultiplet.

The lowest weight vectors of a generic doubleton supermultiplet of spin range 2 and its CPT-conjugate partner are [4,5]

$$|\Omega\rangle = \xi^{A_1}\xi^{A_2}...\xi^{A_{2j}}|0\rangle, \quad |\Omega\rangle = \eta^{A_1}\eta^{A_2}...\eta^{A_{2j}}|0\rangle. \tag{25}$$

VI. "MASSLESS" SUPERMULTIPLETS OF $SU(2,2|4)$

The doubleton supermultiplets described in the last subsection are fundamental in the sense that all other lowest weight UIR's of $SU(2,2|4)$ occur in the tensor product of two or more doubleton supermultiplets. Instead of trying to identify these irreducible submultiplets in the (in general reducible, but not fully reducible) tensor products, one simply increases the number P of oscillator generations so that the tensoring becomes implicit while the irreducibility stays manifest.

The simplest class, corresponding to $P=2$, contains the supermultiplets that are commonly referred to as "massless" in the $5d$ AdS sense. We will therefore use this as a name for all supermultiplets that are obtained by taking $P=2$ in the oscillator construction despite some problems with the notion of "mass" in AdS spacetimes [4].

We will now give a complete list of the allowed $SU(2,2|4)$ lowest weight vectors $|\Omega\rangle$ for $P=2$.

The condition $L^-|\Omega\rangle = 0$ leaves the following possibilities:

- $|\Omega\rangle = |0\rangle$. This lwv gives rise to the $\mathcal{N}=8$ graviton supermultiplet in AdS_5 and occurs in the tensor product of two CPT self-conjugate doubleton (i.e. $\mathcal{N}=4$ super Yang Mills) supermultiplets.
- $|\Omega\rangle = \xi^{A_1}(1)\xi^{A_2}(1)...\xi^{A_{2j}}(1)|0\rangle$. The corresponding supermultiplets and also their conjugates resulting from
- $|\Omega\rangle = \eta^{A_1}(1)\eta^{A_2}(1)...\eta^{A_{2j}}(1)|0\rangle$ have been listed in [4] (Tables 8 to 11). Increasing j leads to multiplets with higher and higher spins and AdS energies. For $j>3/2$ the spin range is always 4. None of these multiplets can occur in the tensor product of two or more self-conjugate doubleton supermultiplets. They require the chiral doubleton supermultiplets.
- $|\Omega\rangle = \xi^{A_1}(1)\xi^{A_2}(1)...\xi^{A_{2j_L}}(1)\eta^{B_1}(2)\eta^{B_2}(2)...\eta^{B_{2j_R}}(2)|0\rangle$. The corresponding supermultiplets have been listed in [4] (Table 12). Again they involve spins and AdS energies that increase with j_L and j_R, maintaining a constant spin range of 4 for $j_L, j_R \geq 1$.

In addition to these purely (super)symmetrized lwv's, one can also anti-(super)symmetrize pairs of superoscillators, since $P=2$. The requirement $L^-|\Omega\rangle = 0$ then rules out the simultaneous appearance of ξ's and η's so that one is left with

- $|\Omega\rangle = \xi^{[A_1}(1)\xi^{B_1]}(2)...\xi^{[A_n}(1)\xi^{B_n]}(2)\xi^{C_1}(1)\ldots\xi^{C_k}(1)|0\rangle$
- $|\Omega\rangle = \eta^{[A_1}(1)\eta^{B_1]}(2)...\eta^{[A_n}(1)\eta^{B_n]}(2)\eta^{C_1}(1)\ldots\eta^{C_k}(1)|0\rangle$.

The special case $k=0$ then leads to the novel short multiplets listed in [5]. The simplest case is given by the following lowest weight vectors

$$|\Omega\rangle = \xi^{[A_1}(1)\xi^{B_1]}(2)|0\rangle, \quad |\Omega\rangle = \eta^{[A_1}(1)\eta^{B_1]}(2)|0\rangle \tag{26}$$

describing a supermultiplet of spin range 2 and its CPT conjugate supermultiplet, respectively. Acting on $|\Omega\rangle$ with the supersymmetry generators $\vec{a}^i \cdot \vec{\beta}^x$ and $\vec{b}^r \cdot \vec{\alpha}^\gamma$ of L^+ and collecting resulting $SU(2,2)\times SU(4)$ lwv's (i.e. states that are annihilated by $\vec{a}_i \cdot \vec{b}_r$ and $\vec{\alpha}_\gamma \cdot \vec{\beta}_x$), one arrives at the supermultiplet of spin range 2 . These supermultiplets do not occur in the tensor product of two or more CPT self-conjugate doubleton supermultiplets, but they appear in the tensor product of two doubleton supermultiplets of spin 3/2.

The general lwvs for $j \geq 2$ [5]

$$|\Omega\rangle = \xi^{[A_1}(1)\xi^{B_1]}(2)...\xi^{[A_j}(1)\xi^{B_j]}(2)|0\rangle, \quad |\Omega\rangle = \eta^{[A_1}(1)\eta^{B_1]}(2)...\eta^{[A_j}(1)\eta^{B_j]}(2)|0\rangle \tag{27}$$

lead to a supermultiplet with spin range 2 and its CPT conjugate partner. Obviously, the spin content of these multiplets is independent of j. Only the AdS energies (conformal dimensions) get shifted, when j is increased, which distinguishes these multiplets from their (super)symmetrized counterparts obtained from $|\Omega\rangle = \xi^{A_1}(1)\xi^{A_2}(1)...\xi^{A_{2j}}(1)|0\rangle$, where the spins increase with j.

VII. CONCLUSIONS

In this talk we have reviewed the unitary supermultiplets of the $\mathcal{N}=8$ $d=5$ anti-de Sitter superalgebra $SU(2,2|4)$ [4,5] and given a complete classification of the doubleton supermultiplets of $SU(2,2|4)$. The doubleton supermultiplets do not have a smooth Poincaré limit in $d=5$. They correspond to $d=4$ superconformal field theories living on the boundary of AdS_5, where $SU(2,2|4)$ acts as the $\mathcal{N}=4$ extended superconformal algebra. The unique CPT self-conjugate irreducible doubleton supermultiplet is simply the $\mathcal{N}=4$ super Yang-Mills multiplet in $d=4$ [12]. However, there are also chiral (i.e. non-CPT self-conjugate) doubleton supermultiplets with higher spins. The maximum spin range of the general doubleton supermultiplet is 2. We have also reviewed the supermultiplets of $SU(2,2|4)$ that can be obtained by tensoring two doubleton supermultiplets. This class of supermultiplets has a maximal spin range of four and contains the multiplets that are commonly referred to as "massless" in the $5d$ AdS sense including the "massless" $\mathcal{N}=8$ graviton supermultiplet in AdS_5 with spin range two. Some of these supermultiplets were studied recently [21] using the language of $\mathcal{N}=4$ conformal superfields developed sometime ago [22]. We have pointed out that there exist some novel short supermultiplets of $SU(2,2|4)$ that have spin range two and do not appear in the Kaluza-Klein spectrum of IIB supergravity. These novel short supermultiplets do not occur in tensor products of the $\mathcal{N}=4$ Yang-Mills supermultiplet with itself, but they can be obtained by tensoring higher spin chiral doubleton supermultiplets. Both kinds of "massless" supermultiplets should be realized in the spectrum of type IIB string theory on $AdS_5 \times S^5$ [6].

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