

# SUSY Q-balls as Dark Matter

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Stable baryonic Q-balls are natural dark-matter candidates in supersymmetric extensions of the Standard Model. Both ordinary matter and dark matter can be produced from a fragmentation of the Affleck-Dine condensate in the early Universe.

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## I. INTRODUCTION

Non-topological solitons associated with some conserved global charge (Q-balls) appear in scalar field theories that have some “attractive” interactions [1,2]. It was recently shown [3] that supersymmetric generalizations of the standard model, in particular the MSSM, contain such solitons in their spectrum. The role of the global symmetry in this case is taken by the U(1) symmetry associated with the conservation of the baryon or lepton number. Even more remarkable is the fact that some of the Q-balls can be entirely stable because their mass is less than that of a collection of nucleons with the same baryon number [4].

At the end of inflation in the early universe, the scalar fields develop a large VEV along the flat directions of the scalar potential. The condensate may carry some baryon or lepton number, in which case it can be thought of as Q-matter, or a superhorizon-size Q-ball. The subsequent evolution of this condensate may give rise to the baryon asymmetry of the universe [5–7]. However, a common assumption that the condensate remains spatially homogeneous fails in many cases. In fact, an initially homogeneous solution of the equations of motion may become unstable with respect to small coordinate-dependent perturbations [4]. The exponentially fast growth of these perturbations can lead to a fragmentation of the scalar condensate with global charge into separate Q-balls. Very large stable baryonic Q-balls (B-balls) can be produced this way.

## II. Q-BALLS

For a simple example, let us consider a field theory with a scalar potential  $U(\varphi)$  that has a global minimum  $U(0) = 0$  at  $\varphi = 0$ . Let  $U(\varphi)$  have an unbroken global<sup>1</sup> U(1) symmetry at the origin,  $\varphi = 0$ . And let the scalar field  $\varphi$  have a unit charge with respect to this U(1).

The charge of some field configuration  $\varphi(x, t)$  is  $Q = \frac{1}{2i} \int \varphi^* \overleftrightarrow{\partial}_t \varphi d^3x$ . Since a trivial configuration  $\varphi(x) \equiv 0$  has zero charge, the solution that minimizes the energy,

$$E = \int d^3x \left[ \frac{1}{2} |\dot{\varphi}|^2 + \frac{1}{2} |\nabla \varphi|^2 + U(\varphi) \right], \quad (1)$$

and has a given charge  $Q > 0$ , must differ from zero in some (finite) domain. This is a Q-ball. It is a time-dependent solution, more precisely it has a time-dependent phase. However, all physical quantities are time-independent. Of course, we have not proven that such a “lump” is finite, or that it has a lesser energy than the collection of free particles with the same charge; neither is true for a general potential. A finite-size Q-ball is a minimum of energy and is stable with respect to decay into free  $\varphi$ -particles if

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<sup>1</sup>Q-balls associated with a local symmetry have been constructed [8]. An important qualitative difference is that, in the case of a local symmetry, there is an upper limit on the charge of a stable Q-ball. Non-abelian symmetries also give rise to Q-balls [9].

$$U(\varphi) / \varphi^2 = \min, \quad \text{for } \varphi = \varphi_0 > 0. \quad (2)$$

One can show that the equations of motion for a Q-ball in 3+1 dimensions are equivalent to those for the bounce associated with tunneling in 3 Euclidean dimensions in an effective potential  $\hat{U}_\omega(\varphi) = U(\varphi) - (1/2)\omega^2\varphi^2$ , where  $\omega$  is such that it extremizes [10]

$$\mathcal{E}_\omega = S_3(\omega) + \omega Q. \quad (3)$$

Here  $S_3(\omega)$  is the three-dimensional Euclidean action of the bounce in the potential  $\hat{U}_\omega(\varphi)$ . The Q-ball solution has the form  $\varphi(x, t) = e^{i\omega t}\bar{\varphi}(x)$ , where  $\bar{\varphi}(x)$  is the bounce.

The analogy with tunneling clarifies the meaning of condition (2), which simply requires that there exist a value of  $\omega$ , for which  $\hat{U}_\omega(\varphi)$  is negative for some value of  $\varphi = \varphi_0 \neq 0$  separated from the false vacuum by a barrier. This condition ensures the existence of a bounce. (Clearly, the bounce does not exist if  $\hat{U}_\omega(\varphi) \geq 0$  for all  $\varphi$  because there is nowhere to tunnel.)

In the true vacuum, there is a minimal value  $\omega_0$ , so that only for  $\omega > \omega_0$ ,  $\hat{U}_\omega(\varphi)$  is somewhere negative. If one considers a Q-ball in a metastable false vacuum, then  $\omega_0 = 0$ . The mass of the  $\varphi$  particle is the upper bound on  $\omega$  in either case. Large values of  $\omega$  correspond to small charges [10]. As  $Q \rightarrow \infty$ ,  $\omega \rightarrow \omega_0$ . In this case, the effective potential  $\hat{U}_\omega(\varphi)$  has two nearly-degenerate minima; and one can apply the thin-wall approximation to calculate the Q-ball energy [2]. For smaller charges, the thin-wall approximation breaks down, and one has to resort to other methods [10].

The above discussion can be generalized to the case of several fields,  $\varphi_k$ , with different charges,  $q_k$  [3]. Then the Q-ball is a solution of the form

$$\varphi_k(x, t) = e^{iq_k\omega t}\varphi_k(x), \quad (4)$$

where  $\varphi(x)$  is again a three-dimensional bounce associated with tunneling in the potential

$$\hat{U}_\omega(\varphi) = U(\varphi) - \frac{1}{2}\omega^2 \sum_k q_k^2 |\varphi_k|^2. \quad (5)$$

As before, the value of  $\omega$  is found by minimizing  $\mathcal{E}_\omega$  in equation (3). The bounce, and, therefore, the Q-ball, exists if

$$\mu^2 = 2U(\varphi) \bigg/ \left( \sum_k q_k \varphi_{k,0}^2 \right) = \min, \\ \text{for } |\vec{\varphi}_0|^2 > 0. \quad (6)$$

The soliton mass can be calculated by extremizing  $\mathcal{E}_\omega$  in equation (3). If  $|\vec{\varphi}_0|^2$  defined by equation (6) is finite, then the mass of a soliton  $M(Q)$  is proportional to the first power of  $Q$ :

$$M(Q) = \tilde{\mu}Q, \quad \text{if } |\vec{\varphi}_0|^2 \neq \infty. \quad (7)$$

In particular, if  $Q \rightarrow \infty$ ,  $\tilde{\mu} \rightarrow \mu$  (thin-wall limit) [1,2]. For smaller values of  $Q$ ,  $\tilde{\mu}$  was computed in [10]. In any case,  $\tilde{\mu}$  is less than the mass of the  $\phi$  particle by definition (6).

However, if the scalar potential grows slower than the second power of  $\phi$ , then  $|\vec{\varphi}_0|^2 = \infty$ , and the Q-ball never reaches the thin-wall regime, even if  $Q$  is large. The value of  $\phi$  inside the soliton extends as far as the gradient terms allow, and the mass of a Q-ball is proportional to  $Q^p$ ,  $p < 1$ . In particular, if the scalar potential has a flat plateau  $U(\phi) \sim m$  at large  $\phi$ , then the mass of a Q-ball is [26]

$$M(Q) \sim mQ^{3/4}. \quad (8)$$

This is the case for the stable baryonic Q-balls in the MSSM discussed below.

### III. SUPERBALLS IN THE MSSM

The presence of the scalar fields with conserved global charges and the requisite “attractive” interactions allows for the existence of Q-balls in the supersymmetric extensions of the standard model. Superpartners of quarks and leptons carry the baryon and the lepton numbers that play the role of charge  $Q$  discussed above.

There are two different sources of the attractive scalar self-interaction that satisfy the criterion (6). First, the tri-linear couplings arise from the superpotential

$$W = yH_2\Phi\phi + \tilde{\mu}H_1H_2 + \dots \quad (9)$$

as well as from supersymmetry breaking terms. Here  $\Phi$  stands for either a left-handed quark ( $\tilde{Q}_L$ ), or a lepton ( $\tilde{L}_L$ ) superfield, and  $\phi$  denotes the right-handed  $\tilde{q}_R$  or  $\tilde{l}_R$ , respectively. The corresponding scalar potential must, therefore, have cubic terms of the form  $y\tilde{\mu}H_2\Phi\phi$ . In addition, there are soft supersymmetry breaking terms of the form  $yAH_1\Phi\phi$ . The condition (6) is automatically satisfied unless some Yukawa couplings and some soft supersymmetry breaking terms are set to zero [3]. Therefore, Q-balls associated with baryon (B) and lepton (L) number conservation are generically present in the MSSM. The Q-balls associated with the trilinear couplings are generally unstable and decay into fermions, quarks and leptons, in a way similar to that discussed in Ref. [14].

Another source of “attraction” that makes the condition (6) possible is the presence of flat directions in the MSSM. These are lifted by supersymmetry breaking terms. The transmission of the supersymmetry breaking to the observable sector is due to some messenger interaction with a typical scale  $M_s$ . In the absence of detailed understanding of the origin of supersymmetry breaking, I treat the scale  $M_s$  as a phenomenological parameter that can be as low as several TeV (in gauge-mediated scenarios, for example), or as high as the Planck scale. In what follows we will concentrate on the case in which  $M_s$  is below the scalar VEV in a Q-ball. This allows for stable baryonic Q-balls (B-balls) in the MSSM. Some further extensions of MSSM possess stable non-baryonic Q-balls [27].

In addition to global charges, the same scalars carry some gauge charges as well. The gauge structure of Q-balls is discussed in Ref. [11].

The mass of a Q-ball with a scalar VEV that extends beyond  $M_s$  along some flat direction is determined by formula (8). If the condensate has a non-zero baryon number, the mass per unit baryon number decreases with  $Q_B$ , the baryon number of a B-ball:

$$\frac{M(Q_B)}{Q_B} = \frac{m}{Q_B^{1/4}} < 1 \text{ GeV} \quad \text{for } Q_B > 10^{12} \left(\frac{m}{1 \text{ TeV}}\right)^4. \quad (10)$$

A B-ball with a baryon number  $Q_B > 10^{12}$  is entirely stable because it is lighter than a collection of neutrons and protons with the same baryon number.

If such large B-balls have formed in the early universe, they would presently exist as a form of dark matter.

In the early Universe, Q-balls can be created in the course of a phase transition (“solitogenesis”), or they can be produced via fusion in a process reminiscent of the big bang nucleosynthesis (“solitosynthesis”) [21]. However, it is unlikely that either of these processes could lead to a formation of solitons with such an enormous charge. Very large Q-balls can form, however, from the breakdown of a primordial scalar condensate [4] that forms naturally at the end of inflation and is the key element of the Affleck–Dine baryogenesis.

### IV. RELIC Q-BALLS FROM FRAGMENTATION OF THE AFFLECK–DINE CONDENSATE

At the end of inflation, the scalar fields acquire large expectation values along the flat directions. Evolution of a scalar condensate carrying a baryon or lepton number has been studied extensively in connection with the Affleck–Dine scenario for baryogenesis in the MSSM [5]. However, a commonly made assumption that an initially spatially-homogeneous condensate remains homogeneous throughout its evolution turns out to be wrong [4]. In fact, the

condensate often develops an instability with respect to small  $x$ -dependent perturbations that lead to fragmentation of the condensate into Q-balls with the same types of global charges.

Indeed, the baryonic condensate of the form  $\phi = e^{i\omega t}\phi_0$  is nothing but Q-matter, or a universe filled with a Q-ball of infinite size. In a static universe, such field configurations are known to break up into finite-size Q-balls under some conditions [28]. The expansion of the universe makes the analyses more complicated.

One can analyze the stability of a given slowly varying solution  $\phi = R(t)e^{i\Omega(t)}$  (where  $R$  and  $\Omega$  are both real) of the equations of motion with a scalar potential  $U(\phi)$  by adding a small space-dependent perturbation  $\delta R$ ,  $\delta\Omega \propto e^{S(t)-i\tilde{k}\vec{x}}$ . Then one can look for growing modes,  $\text{Re}\alpha > 0$ , where  $\alpha = dS/dt$ . The value of  $k$  is the spectral index in the comoving frame and is red-shifted with respect to the physical wavenumber in the expanding background:  $\tilde{k} = k/a(t)$ , where  $a(t)$  is the scale factor.

Of course, if the instability develops, the linear approximation soon ceases to be valid. However, we assume that the wavelength of the fastest-growing mode sets the scale for the high and low density domains that collapse into Q-ball. This assumption can be verified *post factum* by comparison with a numerical analysis, in which both large and small perturbations are taken into account.

From the equations of motion one can derive a dispersion relation that defines the band of unstable modes,  $0 < k < k_{max}$ , where  $k_{max}(t) = a(t)\sqrt{\tilde{\Omega}^2 - U''(R)}$ . The amplification of a given mode  $k$  is characterized by the exponential of  $S(k) = \int \alpha(k, t)dt$ , and depends on how long the mode remains in the band of instability before (and if) it is red-shifted away from the amplification region.

It is natural to identify the best-amplified mode (that with maximal  $S(k)$ ) with the size of a Q-ball formed as a fragment of the initial condensate.

The detailed analyses of fragmentation for some potentials can be found in Refs. [4,12].

Both the ordinary baryonic matter and the stable B-balls can be produced from a single primordial scalar condensate. Stable baryonic Q-balls make a natural candidate for cold dark matter in theories with supersymmetry if inflation took place in the early universe. This scenario is particularly appealing because, since the dark matter and the ordinary matter are produced in the same process, their amounts are naturally related and are calculable in a given model. Conceivably, the cold dark matter in the Universe can be made up entirely of superballs. Since the baryonic matter and the dark matter share the same origin in the scenario described in the previous section, their contributions to the mass density of the Universe are related. Therefore, one may hope to understand why the observations find  $\Omega_{DARK} \sim \Omega_{nucleon}$  within an order of magnitude. This fact is extremely difficult to explain in models that invoke a dark-matter candidate whose present-day abundance is determined by the process of freeze-out, independent of baryogenesis. If this is the case, one could expect  $\Omega_{DARK}$  and  $\Omega_{nucleon}$  to be different by many orders of magnitude. If one doesn't want to accept this equality as fortuitous, one is forced to hypothesize some *ad hoc* symmetries [20] that could relate the two quantities. In the MSSM with AD baryogenesis, the amounts of dark-matter Q-balls and the ordinary matter baryons are naturally related [4,19]. One predicts [19]  $\Omega_{DARK} = \Omega_{nucleon}$  for B-balls with

$$Q_B \sim 10^{26} \left( \frac{m}{1 \text{ TeV}} \right)^2. \quad (11)$$

A different scenario that relates the amounts of baryonic and dark matter in the Universe, and in which the dark-matter particles are produced from the decay of unstable B-balls was proposed by Enqvist and McDonald [12,15–17].

## V. DETECTION OF PRIMORDIAL SUPERBALLS

Interactions of the superballs with matter [26,13] are determined by the structure of the scalar condensate inside the Q-ball. In the interior of a B-ball the squarks have a large VEV and, therefore, the color SU(3) symmetry is spontaneously broken (the Higgs phase). The flat direction may contain the sleptons and the Higgs fields in addition to the squarks. When a nucleon enters a Q-ball, it dissociates into quarks, and the 1 GeV binding energy is released in soft pions. Then quarks are absorbed into the condensate. Likewise, the electrons can be absorbed by a condensate

in a  $(B - L)$ -ball, for example. A Q-ball that absorbs protons and electrons at roughly the same rate would catalyze numerous proton decays on its passage through matter.

However, the electrons cannot penetrate inside those Q-balls, whose scalar VEV gives them a large mass. For example, the simultaneously large VEV's of both the left-handed ( $L_e$ ) and the right-handed ( $e$ ) selectrons along the flat direction give rise to a large electron mass through mixing with the gauginos. The locked out electrons can form bound states in the Coulomb field of the (now electrically charged) soliton. The resulting system is similar to an atom with an enormously heavy nucleus. Based on their ability to retain electric charge<sup>2</sup>, the relic solitons can be separated in two classes: Supersymmetric Electrically Neutral Solitons (SENS) and Supersymmetric Electrically Charged Solitons (SECS). The interactions of Q-balls with matter, and, hence, the modes of their detection, differ depending on whether the dark matter comprises SENS or SECS.

First, the Coulomb barrier can prevent the absorption of the incoming nuclei by SECS. A Q-ball with baryon number  $Q_B$  and electric charge  $Z_Q$  cannot imbibe protons moving with velocity  $v \sim 10^{-3}c$  if  $Q_B \lesssim 10^{29} Z_Q^4 (m/1 \text{ TeV})^4$ . Second, the scattering cross-section of an electrically charged Q-ball passing through matter is now determined by, roughly, the Bohr's radius, rather than the Q-ball size:  $\sigma \sim \pi r_B^2 \sim 10^{-16} \text{ cm}^2$ .

By numerical coincidence, the total energy released per unit length of the track in the medium of density  $\rho$  is, roughly, the same for SENS and SECS,  $dE/dl \sim 100 (\rho/1 \text{ g cm}^{-3}) \text{ GeV/cm}$ . However, the former takes in nuclei and emits pions, while the latter dissipates its energy in collisions with the matter atoms.

The overall features of the superball track are similar to those of the rare Pamir event described in Ref. [29]. (An optimist may consider this a candidate event.)

Assuming that superballs make up an order-one fraction of dark matter, one can predict the number density

$$n_Q \sim \frac{\rho_{DM}}{M_Q} \sim 5 \times 10^{-5} Q_B^{-3/4} \left( \frac{1 \text{ TeV}}{m} \right) \text{ cm}^{-3}. \quad (12)$$

and the flux  $F \simeq (1/4\pi)n_Q v \sim 10^2 Q_B^{-3/4} \frac{1 \text{ TeV}}{m} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$  of the dark-matter Q-balls [13]. Given the predicted size of dark-matter superballs (11), a passage of a Q-ball through any of the presently operating detectors would be a very rare event. For example, for  $Q_B \sim 10^{26}$  and  $m \sim 1 \text{ TeV}$ , Super-Kamiokande would see one event in a hundred years. Of course, smaller Q-balls with baryon numbers  $10^{22} \dots 10^{24}$  may be detected.

Non-observation of superballs sets the limit on their baryon number (assuming  $\Omega_Q \sim 1$ ). The present limits on SECS comes from the MACRO search for "nuclearites" [30], which have similar interactions with matter:  $F < 1.1 \times 10^{-14} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ . This translates into the lower limit on the baryon number of dark-matter Q-balls,  $Q_B \gtrsim 10^{21}$ . Signatures of SENS are similar to those expected from the Grand Unified monopoles that catalyze the proton decay. If one translates the current experimental limits from Baikal [22] on the monopole flux, one can set a limit on the charge of SENS,  $Q_B \gtrsim 3 \times 10^{22}$ , for  $m = 1 \text{ TeV}$ . Non-observation of Q-balls at the Super-Kamiokande after a year of running would improve this limit by two orders of magnitude. Of course, this does not preclude the existence of smaller Q-balls with lower abundances that give negligible contribution to the matter density of the universe.

Electrically charged Q-balls with a smaller baryon number can dissipate energy so efficiently that they may never reach the detector. SECS with baryon number  $Q_B \lesssim 10^{13} (m/1 \text{ TeV})^{-4/3}$  can be stopped by the 1000 m of water equivalent matter shielding. Such solitons could not have been observed by the underground detectors. Therefore, in the window of  $Q_B \sim 10^{12} \dots 10^{13}$  the flux of SECS appears to be virtually unconstrained.

The present limits will be improved by the future experiments, for example, AMANDA, ANTARES, BAIKAL, and others. A low-sensitivity but large-area (several square kilometers) detector could cover the entire cosmologically interesting range of  $Q_B$ .

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<sup>2</sup>It should be stressed that the condensate inside the Q-ball is electrically neutral (and it is also a singlet with respect to all non-abelian gauge groups) [11]. The electric charge is acquired through interactions with matter [13].

## VI. STAR WRECK: THE SUPERBALL INVASION

Dark-matter superballs pass through the ordinary stars and planets with a negligible change in their velocity. However, both SECS and SENS stop in the neutron stars and accumulate there [23]. As soon as the first Q-ball is captured by a neutron star, it sinks to the center and begins to absorb the baryons into the condensate. High baryon density inside a neutron star makes this absorption very efficient, and the B-ball grows at the rate that increases with time due to the gradual increase in the surface area. After some time, the additional dark-matter Q-balls that fall onto the neutron star make only a negligible contribution to the growth of the central Q-ball [23]. So, the fate of the neutron star is sealed when it captures the first superball.

According to the discussion in section III, the energy per unit baryon number inside the relic B-ball is less than that in nuclear matter. Therefore, the absorption process is accompanied by the emission of heat carried away by neutrinos and photons. We estimate that this heating is too weak to lead to any observable consequences. However, the absorption of nuclear matter by a baryonic Q-ball causes a gradual decrease in the mass of the neutron star.

Neutron stars are stable in some range of masses. In particular, there is a minimal mass (about 0.18 solar mass), below which the force of gravity is not strong enough to prevent the neutrons from decaying into protons and electrons. While the star is being consumed by a superball, its mass gradually decreases, reaching the critical value eventually. At that point, a mini-supernova explosion occurs [24], which can be observable. Perhaps, the observed gamma-ray bursts may originate from an event of this type. A small geometrical size of a neutron star and a large energy release may help reconcile the brightness of the gamma-ray bursts with their short duration.

Depending on the MSSM parameters, the lifetime of a neutron star  $t_s$  can range from 0.01 Gyr to more than 10 Gyr [23]:

$$t_s \sim \frac{1}{\beta} \times \left( \frac{m}{200 \text{ GeV}} \right)^5 \text{ Gyr}, \quad (13)$$

where  $\beta$  is some model-dependent quantity expected to be of order one [23]. The ages of pulsars set the limit  $t_s > 0.1$  Gyr.

It is interesting to note that  $t_s$  depends on the fifth power of the mass parameter  $m$  associated with supersymmetry breaking. If the mini-supernovae are observed (or if the connection with gamma-ray bursts is firmly established), one can set strict constraints on the supersymmetry breaking sector from the rate of neutron star explosions.

The natural billion-year time scale is intriguing.

## VII. CONCLUSION

In conclusion, supersymmetric extensions of the standard model predicts the existence of non-topological solitons, Q-balls. They could be produced in the early universe as a by-product of the Affleck – Dine baryogenesis. Large baryonic superballs can be entirely stable and can contribute to dark matter at present time. This makes superballs a natural candidate for dark matter in theories with low-energy supersymmetry.

Present experimental and astrophysical limits are consistent with superball dark matter. The relic baryonic Q-balls have strong interactions with matter and can be detected in present or future experiments. Observational signatures of the baryonic solitons are characterized by a substantial energy release along a straight track with no attenuation throughout the detector. The present experimental lower bound on the baryon number  $Q_B \gtrsim 10^{22}$  is consistent with theoretical expectations [4] for the cosmologically interesting range of Q-balls in dark matter. In addition, smaller Q-balls, with the abundances much lower than that in equation (12), can be present in the universe. Although their contribution to  $\Omega_{DM}$  is negligible, their detection could help unveil the history of the universe in the early post-inflationary epoch. Since the fragmentation of a coherent scalar condensate [4] is the only conceivable mechanism that could lead to the formation of Q-balls with large global charges, the observation of any Q-balls would seem to speak unambiguously in favor of such process having taken place. This would, in turn, have far-reaching implications

for understanding the origin of the baryon asymmetry of the universe, for the theory of inflation, and for cosmology in general.

The entire cosmologically interesting range of dark-matter superballs could be covered by a detector with a surface area of several square kilometers. Since the required sensitivity is extremely low (thanks to the huge energy release expected from the passage of a superball), it is conceivable that a relatively inexpensive dedicated experiment could perform the exhaustive search and ultimately discover or rule out superballs as dark-matter particles.

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- [1] For review, see T. D. Lee and Y. Pang, Phys. Rept. **221**, 251 (1992).
  - [2] G. Rosen, J. Math. Phys. **9**, 996 (1968); *ibid.* **9**, 999 (1968); R. Friedberg, T. D. Lee, and A. Sirlin, Phys. Rev. **D13**, 2739 (1976); S. Coleman, Nucl. Phys. **B262**, 263 (1985); A. Kusenko, Phys. Lett. **B 404**, 285 (1997).
  - [3] A. Kusenko, Phys. Lett. **B 405**, 108 (1997).
  - [4] A. Kusenko and M. Shaposhnikov, Phys. Lett. **B 418**, 46 (1998).
  - [5] I. Affleck and M. Dine, Nucl. Phys. **B 249**, 361 (1985).
  - [6] M. Dine, L. Randall, and S. Thomas, Phys. Rev. Lett. **75**, 398 (1995).
  - [7] M. Dine, L. Randall, and S. Thomas, Nucl. Phys. **B 458**, 291 (1996); J. A. Casas and G. B. Gelmini, Phys. Lett. **B410**, 36 (1997); B. A. Campbell, M. K. Gaillard, H. Murayama, and K. A. Olive, Nucl.Phys. **B538**, 351 (1999).
  - [8] K. Lee, J. A. Stein-Schabes, R. Watkins and L. M. Widrow, Phys. Rev. **D39** (1989) 1665; T. Shiromizu, Phys. Rev. D58 (1998) 107301; T. Shiromizu, T. Uesugi, M. Aoki, hep-ph/9811420.
  - [9] A. M. Safian, S. Coleman, and M. Axenides, Nucl. Phys. **B297** (1988) 498; M. Axenides, E. Floratos, and A. Kehagias, Phys. Lett. **B444**, 190-195 (1998).
  - [10] A. Kusenko, Phys. Lett. **B 404** (1997) 285.
  - [11] A. Kusenko, M. Shaposhnikov, and P. Tinyakov, Pisma Zh. Eksp. Teor. Fiz. **67**, 229 (1998) [JETP Lett. **67**, 247 (1998)] (hep-th/9801041).
  - [12] K. Enqvist and J. McDonald, Phys. Lett. **B 425**, 309 (1998).
  - [13] A. Kusenko, V. A. Kuzmin, M. Shaposhnikov, and P. G. Tinyakov, Phys. Rev. Lett. **80**, 3185 (1998).
  - [14] A. Cohen, S. Coleman, H. Georgi and A. Manohar, Nucl. Phys. **B272**, 301 (1986).
  - [15] K. Enqvist and J. McDonald, Nucl. Phys. **B538**, 321 (1999).
  - [16] K. Enqvist and J. McDonald, Phys. Rev. Lett. **81** (1998) 3071.
  - [17] K. Enqvist and J. McDonald, hep-ph/9811412.
  - [18] K. Enqvist and J. McDonald, Phys. Lett. **B440** (1998) 59.
  - [19] M. Laine and M. Shaposhnikov, Nucl. Phys. **B532**, 376 (1998).
  - [20] D. B. Kaplan, Phys. Rev. Lett. **68**, 741 (1992).
  - [21] J. A. Frieman, G. B. Gelmini, M. Gleiser and E. W. Kolb, Phys. Rev. Lett. **60**, 2101 (1988); K. Griest, E. W. Kolb and A. Maassarotti, Phys. Rev. **D40**, 3529 (1989); J. Ellis, J. Hagelin, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. **B125**, 275 (1983); K. Griest and E. W. Kolb, Phys. Rev. **D40**, 3231 (1989); J. A. Frieman, A. V. Olinto, M. Gleiser and C. Alcock, Phys. Rev. **D40**, 3241 (1989); A. Kusenko, Phys. Lett. **B 406**, 26 (1997).
  - [22] I. A. Belolaptikov *et al.*, astro-ph/9802223.
  - [23] A. Kusenko, M. Shaposhnikov, P. G. Tinyakov, and I. I. Tkachev, Phys.Lett. **B 423** (1998) 104.
  - [24] M. Colpi, S. L. Shapiro, and S. A. Teukolsky, Astrophys. J. 414 (1993) 717; K. Sumiyoshi, S. Yamada, H. Suzuki, and W. Hillebrandt, astro-ph/9707230.
  - [25] M. Axenides, E. G. Floratos, G. K. Leontaris, and N. D. Tracas, hep-ph/9811371.
  - [26] G. Dvali, A. Kusenko, and M. Shaposhnikov, Phys. Lett. **B 417** (1998) 99.
  - [27] D. A. Demir, Phys.Lett.**B450** (1999) 215.
  - [28] K. Lee, Phys. Rev. **D50** (1994) 5333.
  - [29] S. G. Bayburina *et al.*, Proc. 16th Intern. Cosmic Ray Conf., **7**, 279 (1979); C. M. G. Lattes, Y. Fujimoto, and S. Hasegawa, Phys. Rep. **65** 151 (1980).
  - [30] E. Witten, Phys. Rev. **D 30**, 272 (1984); A. De Rújula and S. L. Glashow, Nature **312**, 734 (1984).