Inflationary Cosmology

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I give an overview of inflationary cosmology.

I. INFLATION IS NATURAL

There are many types of inflation that are natural from the particle physics point of view.

Positive cosmological constant (de Sitter, 1917)

Just needs our vacuum to have positive energy. This type of inflation **never ends** and so cannot be the origin of our hot Big Bang universe.

Observations suggest this is just beginning now. Although a positive cosmological constant is natural, its observed magnitude requires anthropically-selected fine-tuning.

Inflation in the early universe (Gliner, 1969, [1])

Erast Gliner was the first to suggest that inflation could be the origin of our hot Big Bang universe. This idea can be realized in many ways.

False vacuum inflation (Guth, 1980, [2])

Just needs a positive energy false vacuum. This type of inflation has **no clock** to synchronize the end of inflation at spatially separated points and so **cannot** produce our **flat** hot Big Bang universe.

However, this type of inflation probably did happen in the unobservably distant past, solving any initial condition problems inflation might have.

Thermal inflation (Lyth & EDS, 1995, [3])

Just needs a finite temperature effective potential

$$V = V_0 + \left(gT^2 - m^2\right)|\phi|^2 + \ldots\,, \qquad m \ll V_0^{1/4} \tag{1}$$

Inflation occurs as the temperature drops through the range $V_0^{1/4} \gtrsim T \gtrsim m$ when ϕ is held at $\phi = 0$. The temperature acts as the clock that determines the end of inflation. **Not scale-invariant** because $T \propto 1/a$.

Thermal inflation is probably needed to **solve** the **moduli (Polonyi) problem** and has important implications for **baryogenesis** [4] and **dark matter** [5].

Rolling scalar field inflation

Just needs a potential

$$V = V_0 - m^2|\phi|^2 + \ldots\,, \qquad m \sim V_0^{1/2}/M_{\rm Pl} \tag{2}$$

Inflation occurs as the scalar field rolls off the maximum. The scalar field acts as the clock that determines the end of inflation. **Not scale-invariant in general**. ($M_{\rm Pl} = 2.4 \times 10^{18}$ GeV)

Oscillating or rotating inflation? (Damour & Mukhanov, 1997, [6])

Needs a concave potential

$$\frac{dV}{d|\phi|} < \frac{V}{|\phi|} \tag{3}$$

Inflation occurs during the oscillation, or rotation, of the scalar field.

Has been proposed as an alternative to thermal inflation's solution of the moduli problem in the context of gauge-mediated supersymmetry-breaking (Moroi, 1998; Asaka, Kawasaki & Yamaguchi, 1998; [7]).

However, oscillating inflation is extremely **unstable** to the growth of inhomogeneities and decay. Rotating inflation is also generally unstable to the growth of inhomogeneities and the formation of Q-balls.

II. DENSITY PERTURBATIONS

(Starobinsky, 1982, [8])

The most interesting type of inflation is that which is assumed to have produced the density perturbations which are needed to form galaxies, make the patterns observed in the cosmic microwave background radiation, etc.

The density perturbations are generated when small-scale fluctuations (for example vacuum fluctuations) are magnified into large-scale perturbations in the quantities x_i (the clocks of the previous section) that determine the number of e-folds of expansion $N(x)$. This then results in a perturbation in the number of e-folds of expansion δN which in turn generates a curvature/density perturbation (Sasaki & EDS, 1995, [9])

$$\mathcal{R}_{\rm c} = \delta N = \sum_i \frac{\partial N}{\partial x_i} \delta x_i \tag{4}$$

Observations constrain the density perturbations to be approximately **scale-invariant**, corresponding to a spectral index

$$n = 1 \pm 0.2 \tag{5}$$

The only scale-invariant type of inflation is a special case of rolling scalar field inflation called **slow-roll inflation**.

III. SLOW-ROLL INFLATION

(Linde, 1982; Albrecht & Steinhardt, 1982; [10])

This type of inflation is required to produce an approximately scale-invariant spectrum of density perturbations. It is a special case of rolling scalar field inflation, with the stronger condition

$$m \ll \frac{V_0^{1/2}}{M_{\rm Pl}} \tag{6}$$

or, more generally

$$\left(\frac{V'}{V}\right)^2 \ll \frac{1}{M_{\rm Pl}^2} \tag{7}$$

and

$$\left|\frac{V''}{V}\right| \ll \frac{1}{M_{\rm Pl}^2} \tag{8}$$

In this case, assuming a single component inflaton, the spectral index of the density perturbations is given by

$$n \simeq 1 + 2\frac{V'' M_{\rm Pl}^2}{V} - 3\left(\frac{V' M_{\rm Pl}}{V}\right)^2 \tag{9}$$

IV. WHY SLOW-ROLL INFLATION IS DIFFICULT

(EDS, 1994, [11,12])

The conditions for slow-roll inflation are

$$\left(\frac{V'}{V}\right)^2 \ll \frac{1}{M_{\rm Pl}^2} \tag{10}$$

and

$$\left|\frac{V''}{V}\right| \ll \frac{1}{M_{\rm Pl}^2} \tag{11}$$

The first suggests we should be near a maximum, or other extremum, of the potential. The second is clearly non-trivial. For example, many models of inflation are built ignoring gravitational strength interactions, and so are implicitly setting $M_{\rm Pl} = \infty$. Clearly one cannot achieve the second condition in this context.

If the inflationary potential energy is dominated by a supergravity F-term then one can precisely quantify the problem. In this case it is straightforward to show that

$$\frac{V''}{V} = \frac{1}{M_{\rm Pl}^2} + \text{model dependent terms} \tag{12}$$

Thus to build a model of slow-roll inflation one must be able to control the **gravitational strength corrections**.

V. ATTEMPTS AT ACHIEVING SLOW-ROLL INFLATION NATURALLY

One of the better early attempts to naturally achieve a flat inflaton potential was Natural Inflation (Freese, Frieman & Olinto, 1990, [13]). It used an approximate U(1) global symmetry to control the inflaton's mass V''. However, one can **not** use the U(1) global symmetry to enforce

$$\left|\frac{V''}{V}\right| \ll \frac{1}{M_{\rm Pl}^2} \tag{13}$$

because V also vanishes in the limit where the symmetry is exact.

A. Special forms for the Kahler potential

(EDS, 1994, [11,12]; Gaillard, Murayama & Olive, 1995, [14])

Special forms for the Kahler potential, such as

$$K = -\ln\left(T + \bar{T} - |\phi|^2\right) \tag{14}$$

combined with some other conditions can give flat inflaton potentials. One has to have good control of the high energy theory to use this method with confidence though.

B. D-term domination of the inflationary potential energy

(EDS, 1994, [12])[1]

For example, consider the following simple globally supersymmetric model. Taking

$$D = \Lambda^2 - |\psi|^2 + |\chi|^2 \,, \qquad W = \lambda\phi\psi\chi \tag{15}$$

where Λ is a Fayet-Iliopoulos term, gives

$$V = \frac{1}{2}g^2\left(\Lambda^2 - |\psi|^2 + |\chi|^2\right)^2 + \lambda^2\left(|\phi|^2|\psi|^2 + |\phi|^2|\chi|^2 + |\psi|^2|\chi|^2\right) \tag{16}$$

The potential is miminised for $\chi = 0$ giving

$$V = \frac{1}{2}g^2\left(\Lambda^2 - |\psi|^2\right)^2 + \lambda^2|\phi|^2|\psi|^2 \tag{17}$$

which is a hybrid inflation potential (Linde, 1991, [16]). For

$$|\phi| > \frac{g\Lambda}{\lambda} \tag{18}$$

the potential is minimised for $\psi = 0$ giving

$$V = \frac{1}{2}g^2\Lambda^4 \tag{19}$$

Because the inflationary potential energy is D-term dominated, there are no mandatory inflaton dependent supergravity corrections.

Superficially this method is very attractive, one only needs a suitable Fayet-Iliopoulos term. However, that is precisely the problem.

It would be natural to identify the Fayet-Iliopoulos term with that obtained in many string compactifications. In weakly coupled string theory, such a Fayet-Iliopoulos term, and the gauge coupling, are inversely proportional to the dilaton

$$\Lambda^2 \propto g^2 \propto \frac{1}{\mathrm{Re}\, S} \tag{20}$$

making the potential unstable against $\mathrm{Re}\, S \to \infty$. Furthermore, to obtain the correct amplitude for the density perturbations one would require (Binetruy & Dvali, 1996, [17])

$$\Lambda \sim 7 \times 10^{15}\,\mathrm{GeV} \tag{21}$$

To obtain this one would need to stabilise the dilaton at a value

$$\mathrm{Re}\, S \sim 10^3 - 10^5 \tag{22}$$

Further yet, one has to do this at large positive potential energy density and without the aid of F-term supersymmetry breaking.

More generally, for example in M theory or other string theories, one could replace the dilaton by some other modulus. However, the problem remains unchanged. One has to stabilise the modulus at a very large value, and furthermore do this at large positive potential energy density and without the aid of F-term supersymmetry breaking.

An alternative might be to try generating the Fayet-Iliopoulos term in field theory, but unless one can do this without F-term supersymmetry breaking one is just led back to the problem of stopping the inflaton getting too large a mass from F-term supersymmetry breaking.

[1] For an earlier D-term model of inflation with a different motivation see (J. A. Casas and C. Munoz, 1988, [15]).

C. Flattening the inflaton's potential with quantum corrections

(EDS, 1996, [18])

This idea works best in the context of gravity-mediated supersymmetry-breaking which I will henceforth assume. It also requires a small gauge (or yukawa) coupling

$$\alpha \sim 10^{-1} \text{ to } 10^{-2} \tag{23}$$

and a very low scale of supersymmetry breaking

$$F \lesssim e^{-1/\alpha} M_{\rm Pl}^2 \tag{24}$$

This could be regarded as fine-tuning were it not for the fact that experiments suggest that

$$\alpha_{\rm GUT} \sim 0.04 \tag{25}$$

and

$$F \sim 10^{-16} M_{\rm Pl}^2 \tag{26}$$

Here is how it works. ($M_{\rm Pl} = 1$)

Any model of inflation must have a positive potential energy

$$V = V_0 > 0 \tag{27}$$

We have in mind that this is at the scale of the moduli potential $V_0^{1/4} \sim 10^{10.5}\,\text{GeV} \sim 10^{-8}$. This will induce soft supersymmetry-breaking masses squared of order $V_0 \sim \left(10^{2.5}\,\text{GeV}\right)^2$

$$V(\phi) = V_0 \left[1 - A\,|\phi|^2 + \ldots\right] \tag{28}$$

with $|A| \sim 1$. This is our classical potential. It does not give rise to slow-roll inflation. In particular

$$\left|\frac{V''}{V}\right| \simeq |A| \sim 1 \tag{29}$$

Now ϕ's couplings to other fields will renormalise ϕ's mass leading to an effective potential of the form

$$V(\phi) = V_0 \left[1 - f(\epsilon \ln|\phi|)\,|\phi|^2 + \ldots\right] \tag{30}$$

where $0 < \epsilon \ll 1$ is proportional to the gauge or Yukawa coupling.

For clarity, consider first the special case

$$V(\phi) = V_0 \left[1 - A\,(1 + \epsilon \ln|\phi|)\,|\phi|^2 + \ldots\right] \tag{31}$$

Define

$$\phi_* \equiv \exp\left(-\frac{1}{\epsilon}\right) \tag{32}$$

and rewrite the potential as

$$V(\phi) = V_0 \left[1 - \epsilon A \ln\left(\frac{|\phi|}{\phi_*}\right) |\phi|^2 + \ldots\right] \tag{33}$$

Now

$$\frac{1}{\sqrt{2}}\frac{V'}{V_0} = -\epsilon A \left[\ln\left(\frac{|\phi|}{\phi_*}\right) + \frac{1}{2} \right] |\phi| + \dots \tag{34}$$

and

$$\frac{V''}{V_0} = -\epsilon A \left[\ln\left(\frac{|\phi|}{\phi_*}\right) + \frac{3}{2} \right] + \dots \tag{35}$$

Therefore the potential has an extremum at $|\phi| = e^{-1/2}\phi_*$ which is a maximum if $A > 0$, which we henceforth assume. The key point is that although $|V''/V| \sim 1$ over most of the potential, the quantum corrections have flattened the potential in the vicinity of the maximum. **Thus we automatically get slow-roll inflation as ϕ rolls off the maximum.**

More generally, assuming f has a zero in the range $V_0 \ll |\phi|^2 \ll 1$, and defining

$$f_* \equiv f(\epsilon \ln \phi_*) = f(-1) \equiv 0 \tag{36}$$

gives

$$V(\phi) = V_0 \left\{ 1 - \left[\epsilon f'_* \ln\left(\frac{|\phi|}{\phi_*}\right) + \mathcal{O}\left(\epsilon^2 \ln^2 \frac{|\phi|}{\phi_*}\right) \right] |\phi|^2 \right\} \tag{37}$$

$$\frac{1}{\sqrt{2}}\frac{V'}{V_0} = -\left[\epsilon f'_* \ln\left(\frac{|\phi|}{\phi_*}\right) + \frac{1}{2}\epsilon f'_* + \mathcal{O}\left(\epsilon^2 \ln^2 \frac{|\phi|}{\phi_*}\right) \right] |\phi| \tag{38}$$

$$\frac{V''}{V_0} = -\left[\epsilon f'_* \ln\left(\frac{|\phi|}{\phi_*}\right) + \frac{3}{2}\epsilon f'_* + \mathcal{O}\left(\epsilon^2 \ln^2 \frac{|\phi|}{\phi_*}\right) \right] \tag{39}$$

In particular, at the extremum

$$\frac{V''}{V} = -\epsilon f'_* + \mathcal{O}\left(\epsilon^2\right) \tag{40}$$

The COBE observations require

$$\frac{V^{3/2}}{V'} = \frac{V_0^{1/2}}{\sqrt{2}\,\epsilon f'_* \, |\phi| \ln(|\phi| / \phi_*)} = 6 \times 10^{-4} \tag{41}$$

i.e.

$$|\phi| = \frac{1/60}{\sqrt{2}\,\epsilon f'_* \ln(|\phi| / \phi_*)} \times 10^5 \times V_0^{1/2} \tag{42}$$

For $V_0^{1/2} \sim (10^{10.5}\,\mathrm{GeV})^2 \sim 10^{-16}$ this gives

$$|\phi| \sim 10^{-11} \tag{43}$$

Now, the condition for slow-roll is

$$\left| \epsilon \ln \frac{|\phi|}{\phi_*} \right| \ll 1 \tag{44}$$

i.e.

$$|\phi| \sim \phi_* = \exp\left(-\frac{1}{\epsilon}\right) \tag{45}$$

Therefore

$$\epsilon \simeq 0.04 \tag{46}$$

which is consistent with the GUT gauge coupling $\alpha_{\rm GUT} \simeq 0.04$ deduced from the LEP data.

The spectral index is fairly close, but not very close, to one. It has the general form

$$\begin{aligned} n &= 1 - 2\epsilon f'_* + B k^{\epsilon f'_*} && (47) \\ &= 1 - 0.08 f'_* + B k^{0.04 f'_*} \quad \text{for } \epsilon = 0.04 && (48) \end{aligned}$$

which is to be compared with the observational constraint

$$n = 1 \pm 0.2 \tag{49}$$

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