Study of the Decays $K_L^0 \rightarrow \pi^+\pi^-\gamma$ and $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ at KTeV

John Belz
Rutgers University

We present new results on the related rare $K_L^0$ decay modes $K_L^0 \rightarrow \pi^+\pi^-\gamma$ and $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$. KTeV has performed the first direct measurement of the form factor for the “direct emission” component of $K_L^0 \rightarrow \pi^+\pi^-\gamma$ decays, a quantity with ramifications for particular chiral models. In addition, the form factor and direct emission/inner bremsstrahlung branching ratio — also presented here — are important input parameters for the understanding of the planar-angle distribution of $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ decays. Preliminary results indicating the presence of a T-violating asymmetry in the $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ angular distribution are presented.

I. INTRODUCTION

In this report we discuss recent studies of the related rare $K_L^0$ decay modes $K_L^0 \rightarrow \pi^+\pi^-\gamma$ and $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ by the KTeV experiment at Fermilab. The decays share the “Inner Bremsstrahlung” (IB) and “Direct Emission” (DE) diagrams of Figure 1:

![Diagrams](image)

FIG. 1. Contributions to $K_L^0 \rightarrow \pi^+\pi^-\gamma^*$ from (left) Inner Bremsstrahlung, (center) direct emission, and (right) $K^0$ charge radius.

The two decays differ in that the photon converts internally in the $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ case. A third diagram known as the “$K^0$ Charge Radius” contribution contributes at a low level ($\sim 4\%$ of the DE [1]) to the $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ branching ratio only.

One item of particular interest is the form factor for the DE decay. Historically [2] [3] this has been expressed in the form of a $\rho$-propagator modification to the magnetic dipole (M1) DE amplitude:

$$\frac{d\mathcal{F}_{M1}}{dE^\gamma} \sim |\mathcal{F}\mathcal{M}_{M1}|^2$$

(1)

where

$$\mathcal{F} = \frac{a_1}{(m^2_{\rho} - m^2_K)} + \frac{a_2}{2m_K E^\gamma}$$

(2)

The effect of the form factor is to soften the photon energy spectrum of the M1 DE decay. The parametric ratio $a_1/a_2$ can then be related to the DE branching ratio by particular chiral models [2] [4].

The decay $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ which was first reported in the literature by KTeV [5] provides the opportunity for further study of the $K_L^0 \rightarrow \pi^+\pi^-\gamma^*$ vertices through the coupling of the $e^+e^-$ decay plane to the helicity of the photon. Interference between the DE and IB photon helicity states results in a manifestly $T$-violating angular asymmetry between the $\pi^+\pi^-$ and $e^+e^-$ decay planes [1] [6] [7] [8].

II. THE KTEV EXPERIMENT

KTeV consists of 85 collaborators from 12 institutions from the U.S. and Japan [9]. The primary goal of KTeV is the measurement of the direct $CP$-violating parameter $\Re(e'/e)$ with a precision of a part in $10^{-4}$. The detector configuration is shown in Figure 2.
The KTeV charged particle detector consists of four drift chambers: two upstream and two downstream of a dipole analysing magnet. A 3100 crystal CsI calorimeter provides precise energy measurement of photons and pion versus electron particle identification.

The analyses reported here are from data taken by KTeV between the fall of 1996 and the summer of 1997.

III. $K_L^0 \rightarrow \pi^+\pi^-\gamma$ ANALYSIS

The $K_L^0 \rightarrow \pi^+\pi^-\gamma$ events for the analysis presented here were taken from the fall 1996 KTeV data set. This data accounts for roughly a tenth of the $K_L^0 \rightarrow \pi^+\pi^-\gamma$ data in hand. Events were selected by requiring:

- Two high-quality tracks forming a good vertex and satisfying fiducial volume cuts and standard cuts on the veto counters. Energy deposited in the calorimeter must not be consistent with an electron.
- At least one additional cluster of energy greater than 1.5 GeV must be present in the calorimeter.
- The pion and gamma clusters in the calorimeter must be separated by at least 20 centimeters. This is to suppress backgrounds from $K_L^0 \rightarrow \pi^+\pi^- \gamma$ decays accompanied by accidental activity or pion hadronic showers.
- The gamma energy in the center of mass must be greater than 20 MeV.
- The missing $\pi^0$ must have imaginary momentum under the $K_L^0 \rightarrow \pi^+\pi^-\pi^0$ hypothesis in order to suppress this background.

This last requirement is achieved by demanding that the quantity

$$P_{\pi^0}^2 = \frac{(M_K^2 - M_{\pi^0}^2 - M_{\pi\pi}^2)^2 - 4M_{\pi^0}^2 M_{\pi\pi}^2 - 4M_K^2 (P_T^2)_{\pi\pi}}{4((P_T^2)_{\pi\pi} + M_{\pi\pi}^2)}$$

(3)
be negative (< −0.0055 GeV²).

The resultant very clean $K_L^0 \rightarrow \pi^+\pi^-\gamma$ signal is shown in Figure 3.

![Figure 3: KTeV 1996 $\pi^+\pi^-\gamma$ invariant mass distribution, all other cuts have been applied.](image)

The remaining background is small (about 0.5%) and is believed to consist primarily of residual $K_L^0 \rightarrow \pi^+\pi^-$ decays accompanied by pion showers in the calorimeter.

To extract the interesting physics from this decay mode we turn our attention to the photon energy in the kaon center-of-mass system. This distribution for the 1996 data is shown in Figure 4 along with the Monte Carlo distributions for the IB and ($\rho$-propagator form factor modified) DE decays.

![Figure 4: KTeV 1996 $K_L^0 \rightarrow \pi^+\pi^-\gamma$ photon energy in the center-of-mass. Superimposed are the corresponding Monte Carlo IB and ($\rho$-propagator form factor modified) DE decays. MC Normalization and $\rho$-propagator parameter $a_1/a_2$ are obtained from Minuit $\chi^2$ minimization.](image)

The $\rho$-propagator parameter $a_1/a_2$ and the relative normalization

$$f = \frac{\Gamma(DE)}{\Gamma(DE + IB)}$$

(4)
are obtained from Minuit $\chi^2$ minimization. The results of the fit are $a_1/a_2 = -0.729\pm0.026(stat.)$ and $\Gamma(DE)/\Gamma(DE+IB)(E^*_e > 20\text{MeV}) = 0.685 \pm 0.009(stat.)$. Systematic effects in both of these numbers due to backgrounds and accidental activity are negligible as are effects due to the variation of most cuts. 2% systematic errors are assigned to $a_1/a_2$ due to shifts in the result from variation of the lower $E^*_e$ cut. Systematic errors in the ratio $\Gamma(DE)/\Gamma(DE+IB)$ are assigned due to shifts in the result with variations of the $E^*_e$ cut (2%) and $\gamma - \pi$ separation cut (2%).

Results for the form–factor parameter $a_1/a_2$ differ substantially from those reported in the literature [3] as illustrated in Figure 5.

![Figure 5](image_url)

**FIG. 5.** Results of KTeV and previous (FNAL E731) measurements of the $K^0_L \rightarrow \pi^+\pi^-\gamma$ DE form–factor parameter $a_1/a_2$, shown with the theoretical predictions relating $a_1/a_2$ to $\theta \equiv \eta - \eta'$ mixing angle and the SU(3) breaking parameter $\xi$ [4, 2]. $\theta$ can also be related to the DE branching ratio [2], which predicts $\theta \sim -20^\circ \pm 1^\circ$ [3].

The reason for this apparent discrepancy has been traced to the fact [10] that the E731 result assumed the Lin and Valencia model in extracting $a_1/a_2$ whereas the KTeV result is a true model–independent measurement of this quantity. Reanalysis of the E731 data by the author indicates that the data from the two experiments are completely consistent.

**IV. $K^0_L \rightarrow \pi^+\pi^-e^+e^-$ ANALYSIS**

We next turn our attention to the $K^0_L \rightarrow \pi^+\pi^-e^+e^-$ analysis with emphasis on extraction of the $T$–violating angular asymmetry. KTeV reported the first branching ratio measurement for this decay [5] based on a subset of the data and has recently presented an improved number based on the full data set [11] (See Figure 6).
Signal=1811±42 events  
S/B = 40±6  
K   -> πππ (Dalitz)  
Background  

PRL 80, 4123(1998): (3.2±0.4(stat)±0.6(syst))x10^-7  
Vancouver Conf: (3.32±0.14(stat)±0.28(syst))x10^-7

FIG. 6. KTeV $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ signal and branching ratio.

As the acceptance of the KTeV spectrometer for the $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ decay is a strong function of the virtual photon energy the extraction of the angular asymmetry is a two-step process: First the data is fit to extract the best values of the form factor parameter $a_1/a_2$ and the DE amplitude. Then the Monte Carlo is tuned to the best-fit values for the final acceptance calculation.

Figure 7 shows the $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ angular distributions for data and Monte Carlo after the fit for the form factor and M1 amplitude.

FIG. 7. a) $\phi$ and b) $\sin(\phi)\cos(\phi)$ angular distributions. The data are shown as dots, the theoretical expectation (Monte Carlo) as a histogram.

The raw (before acceptance correction) asymmetry of the data is clear. After acceptance correction the value of the asymmetry is:

$$\frac{(N_+ - N_-)}{(N_+ + N_-)} = (14.6 \pm 2.3(stat.))\%$$

where $N_+$ and $N_-$ are the number of events in the right and left hand sides — respectively — of the $\sin(\phi)\cos(\phi)$ plot in Figure 7.

Systematic errors of less than a percent are assigned to the asymmetry from (1) detector resolution (2) uncertainties.
in the form factor and M1 amplitude and (3) variations in physics cuts. The total systematic error on the $K_L^0 \rightarrow \pi^+\pi^-\gamma$ plane–angle asymmetry measurement is estimated to be about 1.1%.

V. SUMMARY AND CONCLUSIONS

The numerical results presented here are summarized in Table 1. Here for the first time we present an improved $K_L^0 \rightarrow \pi^+\pi^-\gamma$ Direct Emission branching ratio. We also present the first theory–independent measurements of a form factor in the $K_L^0 \rightarrow \pi^+\pi^-\gamma^*$ Direct emission process in two complimentary decay modes. The observed planar–angle asymmetry in the $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ decay is the first instance of CP–violation in a dynamical variable and is manifestly $T$–violating as well.

<table>
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